Gravitational Stresses in Deep Beams

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Synopsis

This paper reviews the methods available for the calculation of gravitational stresses in deep beams and presents the results of a series of experiments on such beams using the frozen stress method of photo-elasticity in which gravitational loading was simulated in a large centrifuge. The stresses so obtained are given in a form suitable for use in design and a number of comparisons with theoretical results are shown.

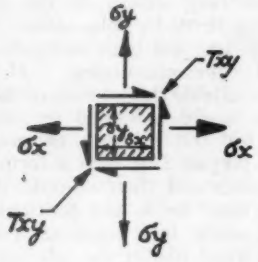

Fig. 1—Notations for stresses and network.

Introduction

As is well known, the ordinary theory of bending cannot be applied to a beam whose depth is greater than about half its span. Such beams, sometimes called "girder walls," are occasionally encountered in structural design and a number of theories have been put forward for the calculation of the stresses in them. When a deep beam forms part of a building structure the self weight of the beam frequently comprises a major proportion of the loading and it is therefore necessary to be able to estimate these gravitational stresses. In principle, this can be done by solution of the bi-harmonic for the Airy stress function :

$$\nabla^4\varphi = \frac{\delta^4\varphi}{\delta x^4} + \frac{2\delta^4\varphi}{\delta x^2\delta y^2} + \frac{\delta^4\varphi}{\delta y^4} = 0$$

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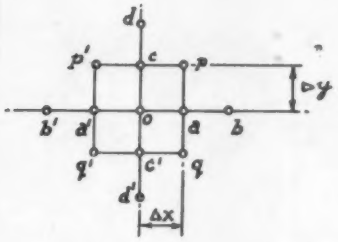


Fig. 2—Notations for stresses and network.

If a solution can be found, the stresses at any point (x, y) will be given by

$$\sigma_x = \frac{\delta^2\varphi}{\delta y^2} - \rho g \cdot y, \quad \sigma_y = \frac{\delta^2\varphi}{\delta x^2} - \rho g \cdot y \quad \text{and}$$

$$T_{xy} = -\frac{\delta^2\varphi}{\delta x \delta y}$$

[σ_x, σ_y and T_{xy} are as shown in Fig. 1 and 'ρg' is the density of the material.]

Unfortunately, in the case of a deep beam there are six boundary conditions to be satisfied in the general loading case and only four constants in the solution of the bi-harmonic so that no general solution is possible. Approximate solutions have, however, been obtained for certain simple loading cases by choosing a particular solution for the bi-harmonic and then finding a polynomial which will cancel out the stresses on the vertical boundaries implicit in the particular solution selected. A solution of this type which includes gravitational stresses has been produced by Durant and Garwood[1]. This solution has the advantage of giving the results as formulae but these are not very rapidly evaluated nor are they very accurate. An alternative approach, capable of giving a greater degree of accuracy, is by the numerical solution of the bi-harmonic by the finite difference method. The disadvantage of this method is the large amount of arithmetical work entailed, although it can be adapted for solution by an electronic computer if one is available.

Although these theoretical methods have been known for some time there does not appear to be any systematic study of the gravitational stress problem in the literature or any corresponding experimental work. The latter fact is probably due to the difficulty of reproducing gravitational loading in the laboratory on model structures. Field observations of body weight stresses are extremely difficult since, in addition

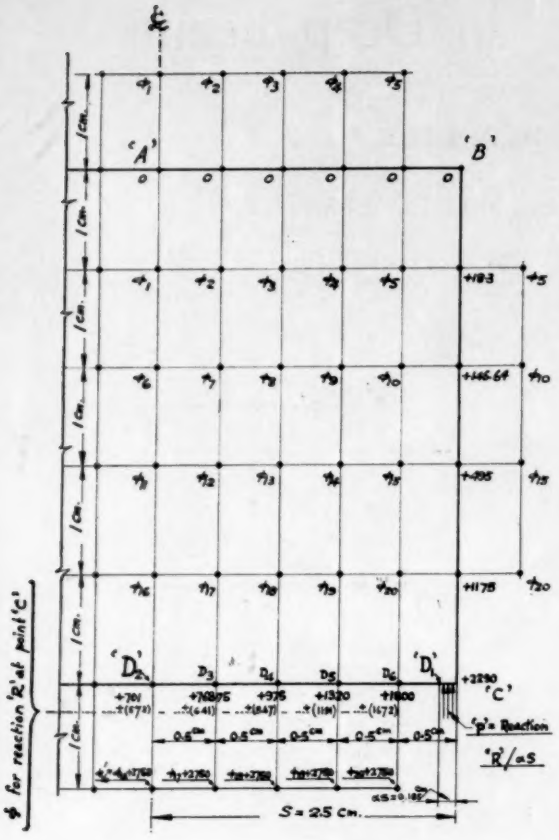


Fig. 3—Mesh for solution by finite differences.

to the usual problems of tests on full sized structures, any gauges intended to measure strains due to this kind of loading must be attached to the structure as it is built.

The authors have overcome the difficulty on the laboratory scale by using a large centrifuge in conjunction with the frozen stress method of photo-elasticity. The equipment used for this method of analysis and the general technique have been described in a paper read at the 50th Anniversary Conference of the Institution of Structural Engineers[2]. In the present experiments the machine produced an acceleration field (at a radius of 54 ins.) equivalent to 100 times that of gravity. The speed of rotation of the centrifuge and the ratio of the model dimensions to the radius of the arm were such that the difference between the centrifugal and true gravitational acceleration fields were negligible; this point is examined in Appendix 1.

Theoretical Analysis of Stresses in Deep Beams

A brief description will be given in the following paragraphs of the method of solution by the formulae of Durant and Garwood and by finite differences. Comparisons with experimental results will be shown later.

(a) Numerical solution by finite differences: The finite difference equation corresponding to the bi-harmonic is, referring to Fig. 2:

$$\varphi_0\left[6\left(\lambda+\frac{1}{\lambda}\right)+8\right]-4\left[(\lambda+1)(\varphi_a+\varphi_{a'})+\left(\frac{1}{\lambda}+1\right)(\varphi_c+\varphi_{c'})\right]+\lambda(\varphi_b+\varphi_{b'})+\frac{1}{\lambda}(\varphi_d+\varphi_{d'})+2(\varphi_p+\varphi_{p'}+\varphi_q+\varphi_{q'})=0$$

[where $\lambda=(\Delta y/\Delta x)^2$]

To illustrate the application of the method, the analysis of one of the test beams whose depth was equal to its span is shown in detail in Appendix II. The dimensions of this beam and the grid adopted for the finite difference solution are shown in Fig. 3.

As will be seen from Appendix II, boundary values are first determined and the values of the stress function at the internal node points are then calculated either by applying the finite difference equation to each of these points in turn and solving the resulting set of simultaneous equations or by assuming arbitrary values at the internal points and improving them by relaxation. Without some kind of computing aid both methods are extremely tedious and time consuming. If an electronic computer is available the solution of the simultaneous equations is easily achieved or, alternatively, a programme for the solution of problems of this type can be prepared in such a form that only the boundary values and their co-ordinates have to be fed into the machine in any particular case. The authors had access to a computer for which a programme was available for the solution of a sufficient number of simultaneous equations; use was therefore made of this programme to evaluate the stress function at the node points of the grid of Fig. 3. Knowing these values the stresses are easily obtained.

(b) Durant and Garwood's Method[1]: Formulae for the stresses σ_x, σ_y and T_{xy} are given in Appendix III. It has been found that rather a large number of terms in the series for the stresses are required and consequently their evaluation is a lengthly procedure.

Experimental Results

Tests were carried out on beams having depth to span ratio '$H:L$' of 0·54 : 1, 1·12 : 1, 1·49 : 1 and 2·23 : 1 using the centrifuge method previously referred to. Fringe photographs obtained in these tests are shown in Fig. 4. The photo-elastic results were reduced by the usual methods, the separate principal stresses being obtained by the numerical solution of the Laplace equation. Magnitudes and directions of these stresses over the beam are shown in Figs. 5, 6, 7 and 8; in order to make the results applicable to any beam of similar depth to span ratio, the values of the stresses are expressed as a fraction or multiple of the maximum extreme fibre stress which would be given by the simple theory of bending. The normal and shear stress distribution across the various sections indicated on Figs. 5 to 8, are shown in Figs. 9, 10, 11 and 12. Statical checks made on these stress distribution diagrams confirmed that the experimental results were correct to within a few per cent.

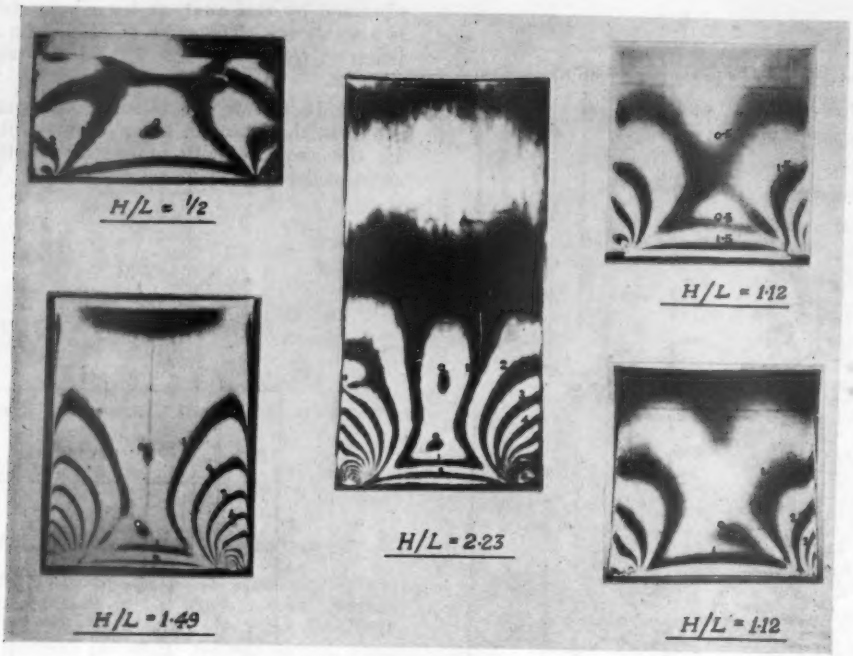


Fig. 4—Fringe patterns in polarized light.

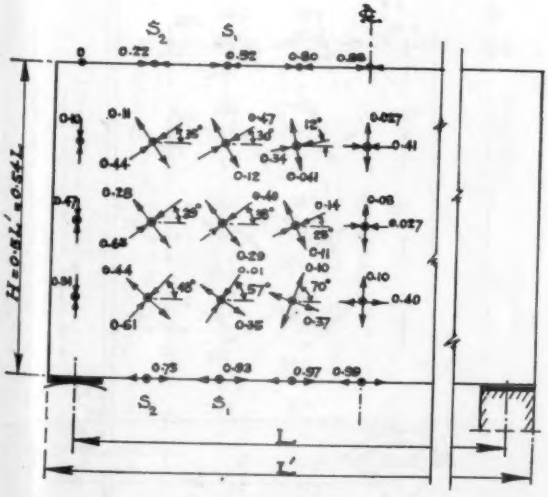


Fig. 5—Principal stresses for a beam H : L = 0.54.

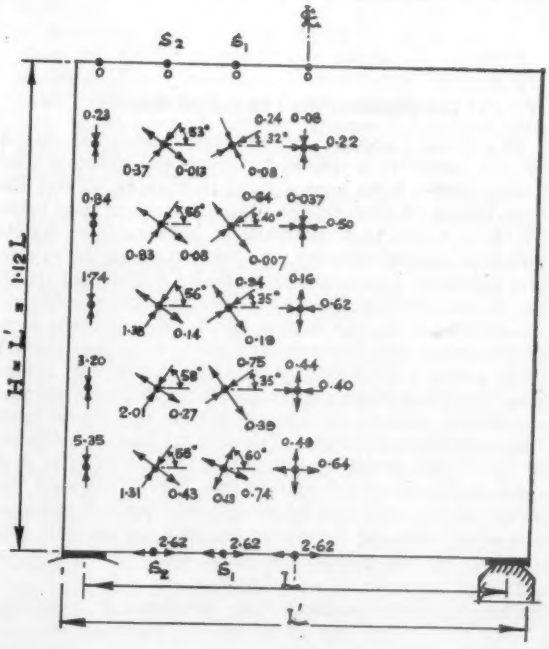


Fig. 6—Principal stresses for a beam H : L = 1.12.

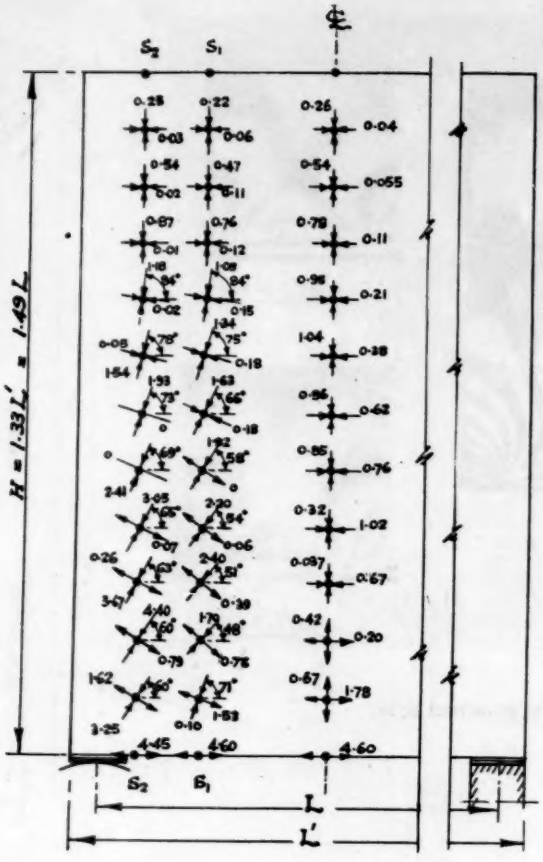


Fig. 7—Principal stresses for a beam H : L = 1.49.

Comparison with Theoretical Results

To provide a comparison with the stresses calculated by the methods mentioned, theoretical stress distribution curves have been plotted in Figs. 10, 12 and 13. Figs. 10 and 13 show for the beam of depth to span ratio ($H : L = 1 \cdot 12 : 1$) a comparison between the distribution of normal stresses on a section along the centre line obtained by experiment and by calculation using the finite difference method. Two support conditions were considered—the first in which the reactions were concentrated and the second in which they were spread over a length of $0 \cdot 037\ L'$ at each end of the beam. The latter case more correctly reflects the experimental conditions and in fact gives better agreement with the experimental results. The straight line distribution of the simple beam theory is also indicated. The grid used in the above calculation (see Fig. 3) required the solution of twenty simultaneous equations. To examine the effect of using a very coarse grid an analysis was carried out in which the mesh size was taken as $\frac{L'}{4} \times \frac{H}{4}$; this required the solution of only six equations and gave the result indicated on Fig. 13.

Fig. 12 gives a similar comparison for the beam of ($H : L = 2 \cdot 23 : 1$) but this time using the solution by Durant and Garwood. The stress distribution obtained by this analysis for a beam of infinite depth is also shown. The same solution was applied for the beam of ($H : L = 1 \cdot 12 : 1$) and the result is given in Fig. 13.

Fig. 14 shows the ratio of the maximum tensile stress in the beam to the maximum fibre stress given by the simple theory of bending plotted against the ratio of depth to span.

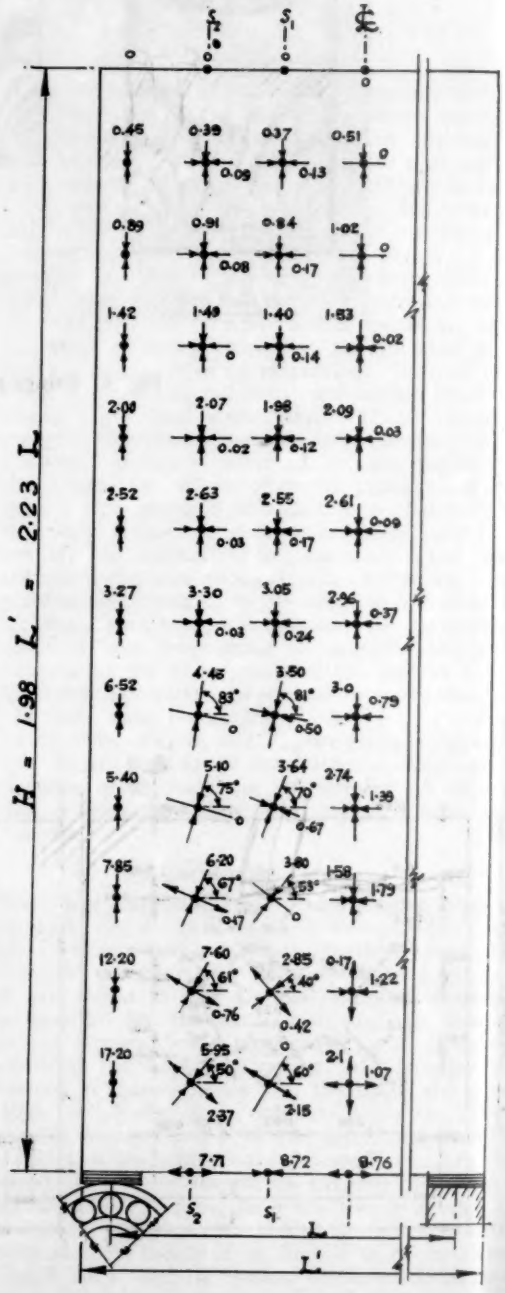


Fig. 8—Principal stresses for a beam H : L = 2.23.

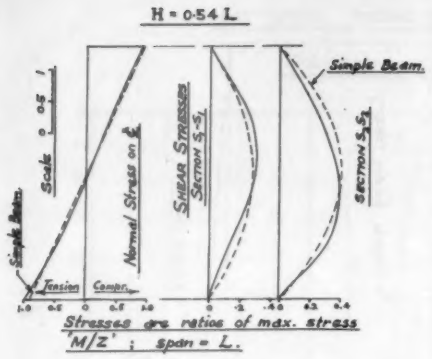


Fig. 9—Normal and shear stresses for a beam H : L = 0.54

Discussion of Results and Conclusions

The results presented in Figs. 10 and 13 confirm that an accurate analysis of the gravitational stresses in a deep beam can be obtained by the finite difference method. The method is easily applied but is time consuming and is unlikely to be of much practical use in structural design unless computing facilities are readily available. Even where these facilities do exist it will not often be worthwhile carrying out a full analysis by the numerical method. The order of magnitude of the stresses can, however, be estimated by using a coarse grid without undue labour.

Durant and Garwood's solution is not very accurate and may be lengthy owing to slow convergence of the series for the stresses but it is of some value in computing approximate maximum tensile stresses in beams whose depth is greater than their span as in this case the approximation of a beam of infinite depth can be employed. An interesting point in this connexion can be seen from the results, namely, that a beam of the proportions just referred to, acts roughly as a beam

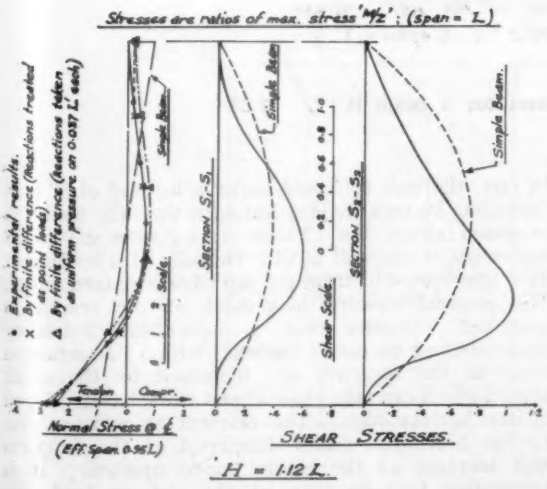

Stresses are ratios of max. stress 'M/Z' ; (span = L)
Experimental results.
By finite differences (Reactions treated as point loads)
By finite difference (reactions taken as uniform pressure on 0·037 L ends)
Simple Beam
Section S₁-S₁
Section S₂-S₂
Shear Scale
Tension
Compr.
Normal Stress @ ℄
(Eff. Span 0·95 L)
SHEAR STRESSES.
H = 1·12 L.

Fig. 10—Normal and shear stresses for a beam H : L = 1.12

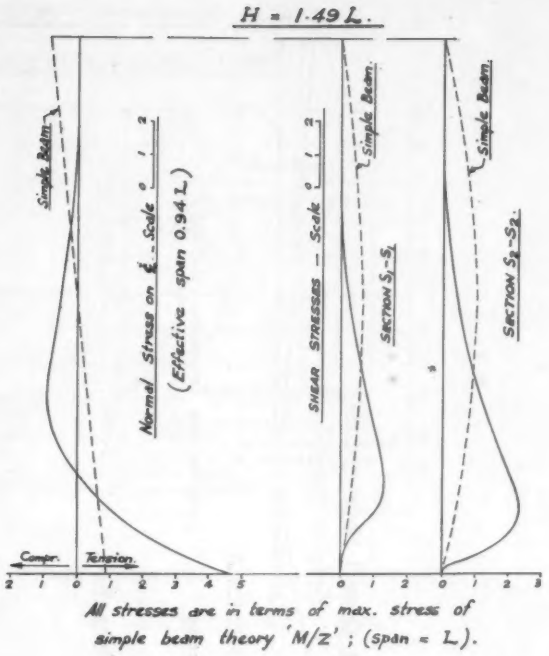

H = 1·49 L.
Simple Beam
Normal Stress on ℄ Scale
(Effective span 0.94 L)
SHEAR STRESSES — Scale
Section S₁-S₁
Section S₂-S₂
Compr.
Tension
All stresses are in terms of max. stress of simple beam theory 'M/Z' ; (span = L).

Fig. 11—Normal and shear stresses for a beam H : L = 1.49

of depth equal to span with the top part merely resting on it. This top portion hardly contributes any resistance to the bending moment due to the dead weight of the beam.

The photo-elastic results presented in this paper will give a sufficiently accurate guide for the design of beams of similar proportions to those tested. If cases are found in which the loading is eccentric or in which there are holes in the beam there is little doubt that photo-elastic analysis will be the most convenient method. It is, however, a method which will be applicable only when the necessary laboratory facilities are available.

In practice it is sometimes necessary to know the stress distribution in a wall beam under its own weight plus a certain live load. This problem arises very often in underpinning schemes and in cases where the strength of a cladding wall is required. Regarding the dead load stresses, which can be of comparable magnitude to the live load stresses in ordinary buildings, a clear picture can be obtained from the diagrams of the principal stresses and for economical design, particularly in reinforced concrete, these diagrams will be helpful in locating the tension zones. The proportioning and placing of reinforcement is more easily achieved by referring to the principal stress diagrams than by adopting the lever arm method which in deep beams can be confusing, as may be seen from reference[3]. It may be relevant to indicate that in beams where the depth to span ratio is less than 1 it is permissible to calculate the maximum stress by considering the dead weight as a uniformly distributed load applied at the top.

Experimental work on deep beams carried out on a 9 inch brick wall by R. H. Wood[4] showed good agreement between the theoretical and experimental loading which would produce the first crack at the

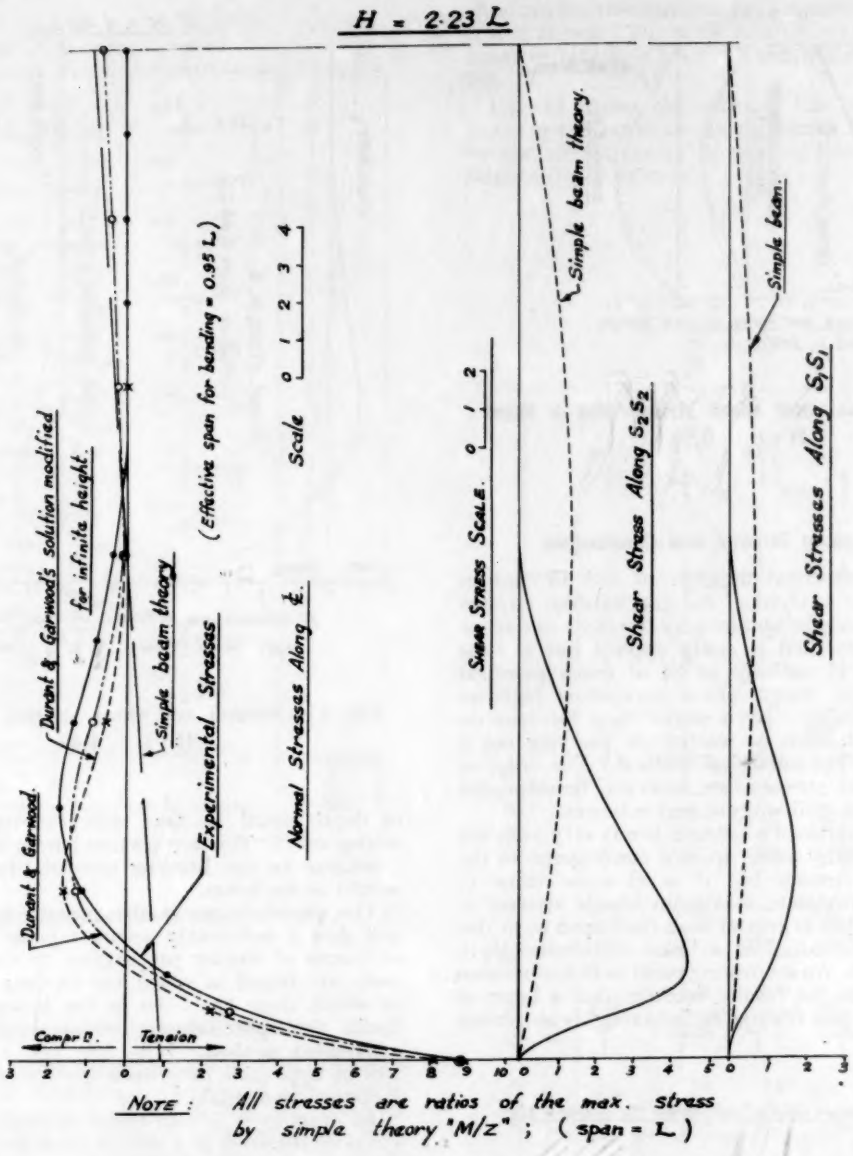

H = 2·23 L
Durant & Garwood's solution modified for infinite height.
Durant & Garwood
Simple beam theory
(Effective span for bending = 0·95 L)
Experimental Stresses
Normal Stresses Along E.
Scale 0 1 2 3 4
Shear Stress Scale. 0 1 2
Simple beam theory.
Shear Stress Along S2S2
Simple beam.
Shear Stresses Along S1S1
Compr'n.
Tension
NOTE: All stresses are ratios of the max. stress by simple theory "M/Z" ; (span = L)

Fig. 12—Normal and shear stresses for a beam H : L = 2.23

bottom edge. Wood[4] and after him Henry[4] in discussing deep beams suggested the existence of an arching effect to account for the high failing load observed in this experiment. It is important to note that such an effect cannot exist unless the conditions at the supports are such as to provide the necessary horizontal restraint. In the test of the 9 inch solid brick wall referred to above, where the clear span and the depth were 10 and 8 ft. respectively, a crack at the bottom edge was developed under a uniformly distributed load of 8 tons applied at the top edge, i.e. under a load of 12 tons inclusive of the dead weight. Although this figure accords very well with the theoretical loading corresponding to a maximum tensile strength of the mortar of 100 lb/in.^{2}, as given in the text, yet failure occurred at a much higher loading simply because of the shear resistance at the supports. In fact, the wall collapsed under a loading of 32 tons (including its own weight) but each time the load was increased above the 12 tons cracks were developed higher up in the wall until eventually at a loading of 31·0 tons only 4 ft. from the top edge remained intact. The gradual upward movement of the cracks as described suggests that a substantial balancing moment along the centre line was built up ; a horizontal shear at the supports was mobilised by the slight spread which can take place after a crack has developed at the bottom edge. The moment at midspan due to the horizontal forces developed at the supports will increase as the cracks move upwards ; it is interesting to note that, at the collapse load, the moment of the maximum horizontal shear at the support about the centre of the effective midspan section (assuming for the mortar a shear strength of 100 lb/in.^{2})

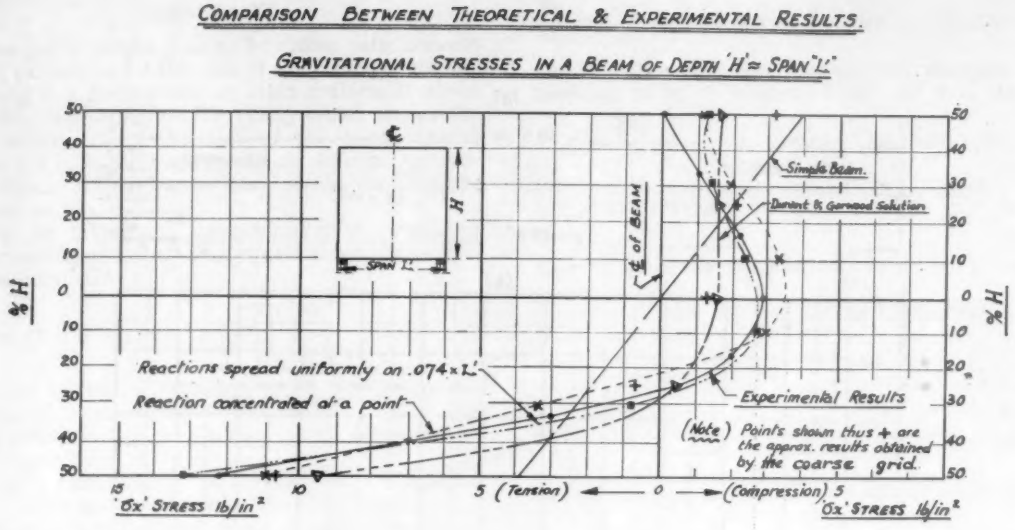

COMPARISON BETWEEN THEORETICAL & EXPERIMENTAL RESULTS.
GRAVITATIONAL STRESSES IN A BEAM OF DEPTH 'H' = SPAN 'L'
Simple Beam.
Durant & Garwood Solution
Reactions spread uniformly on .074 x L'
Reaction concentrated at a point
Experimental Results
(Note) Points shown thus + are the approx. results obtained by the coarse grid.
SPAN 'L'
H
℄ of BEAM
% H
'σx' STRESS lb/in²
5 (Tension)
0
(Compression) 5

Fig. 13—Comparison between experimental and theoretical stresses in a beam H : L = 1.12.

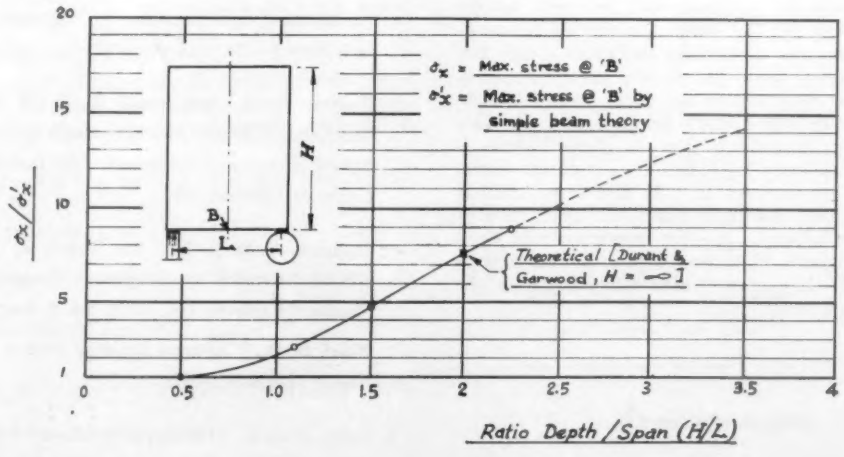

COMPARISON BETWEEN MAXIMUM TENSION BY EXPERIMENT & SIMPLE BEAM THEORY.
σx = Max. stress @ 'B'
σ'x = Max. stress @ 'B' by simple beam theory
Theoretical [Durant & Garwood, H = ∞]
H
B
L
σx/σ'x
Ratio Depth / Span (H/L)

Fig. 14—Showing the validity limit of the simple theory and the quick build up of the error.

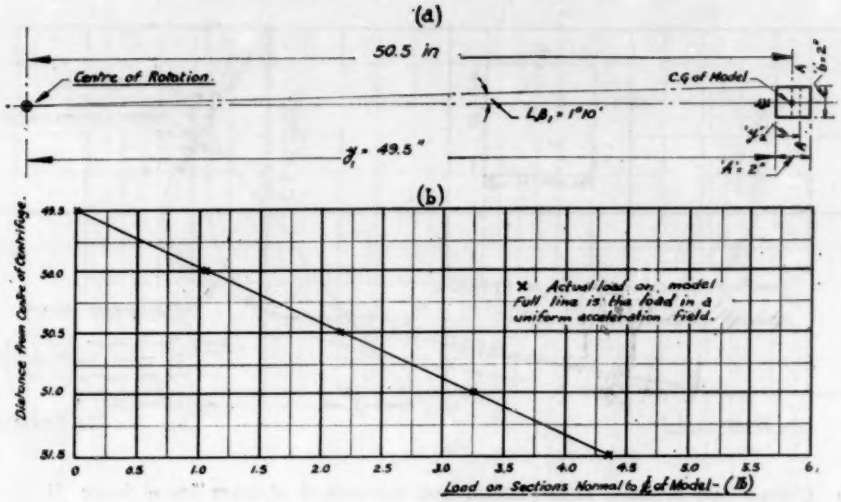


Fig. 15—(a) Dimensions of the model and the field of rotation.

(b) Indicates that the field in the centrifuge approaches very nearly a uniform one.

plus the moment of resistance of the latter are equal to the maximum bending moment induced by that loading. Apparently the supporting blocks were capable of resisting the maximum horizontal shear, which was nearly 4·8 tons, otherwise complete failure would have occurred soon after the 12 ton loading was applied. At the point of failure the ratio of depth to span of the intact portion of the beam was 4 to 10, i.e. the depth was less than half the span and in this case the stress diagram at midspan will be very nearly triangular as given by the simple theory of bending.

The above discussion clearly indicates that the 'arch' action arose from the nature of the support conditions in the experiment and it is not justifiable to assume that it will always be present. To this extent reference to arch action in deep beams is confusing and can only be taken into account if the support conditions warrant it. At the same time it may be noted that secondary horizontal restraints at the supports are of much greater importance in deep beams than in beams of ordinary proportions and as has been shown, they may substantially increase the strength of the member.

Acknowledgement

The work described in this paper was carried out in the Faculty of Engineering, University of Khartoum; thanks are due to the University authorities for the facilities made available. Thanks are also due to Col. H. T. Jessop and Mr. C. Snell of University College, London, for their valuable comments and suggestions on the project.

References

1. Durant, N. J., and Garwood, F., "Ministry of Works Technical Note No. 78," London, October, 1947.

2. Hendry, A. W., and Saad, S., "The Determination of Gravitational Stresses by Photo-elasticity." *Proceedings of the Fiftieth Anniversary Conference of the Institution of Structural Engineers*, October, 1958.

3. Uhlmann, U. H. L. B., "The Theory of Girder Walls with Special Reference to Reinforced Concrete Design," *The Structural Engineer*, Vol. XXX, No. 8, August, 1952.

4. Wood, R. H., "National Building Studies," *Research Paper No.* 13, H.M.S.O. (1952).

5. Henry, F. D. C., "The Design and Construction of Engineering Foundations." E. & F. N. Spon Ltd., London, W.C.2. (pp. 454-456).

Appendix I

Where body weight has to be taken into account Laplace's equation for the sum of the principal stresses holds only in a gravity field or other uniform acceleration field. Referring to Fig. 15(a) which shows the actual conditions in the test of the beam having $H : L = 1 \cdot 12 : 1$, the total weight on different sections perpendicular to the centre line of the model were computed as follows :—

Firstly by taking into consideration the variation in the centrifugal force. The equation for the weight at any section A.A. will be :

$$m \times f^2 \times b \times t \left[y_1 y + \tfrac{1}{2} y^2 \right]_{y=0}^{y=y_2}$$

where 'm' = density (absolute units), 'f' = angular velocity, 't' = thickness. Secondly by assuming a uniform field with an acceleration equal to that at the centre of gravity of the model.

The results are plotted in Fig 15(b) and it is clear that the difference is negligible.

A rigorous solution based on Poisson's equation was obtained for the same beam in order to assess the difference between the correct values of the sum of the principal stresses $(p + q)$ and those obtained from the numerical solution of Laplace's equation. The analysis showed that the order of the correction varied from zero at the boundaries to a maximum of about $\pm 2 \cdot 5$ per cent at the centre of the model. This confirms the impression conveyed by Fig. 15(b) that the acceleration field used in the experiment closely approximated a gravity field and, consequently, that the use of the Laplace equation for the separation of the principal stresses was justified.

Appendix II

It is evident that the stresses derived from the Airy stress function depend on the differences between values of the stress function at the node points and not on the actual values; for example at point 0 in Fig. 2 $\sigma_x = \dfrac{\delta^2 \varphi}{\delta y^2} - \rho g \cdot y = \dfrac{1}{(\Delta y)^2} [\varphi_c - 2\varphi_o + \varphi_c'] - \rho g \cdot y$. Thus any node point on the boundary, say point A on the grid of Fig. 3, may be taken as a datum where $\varphi = \dfrac{\delta \varphi}{\delta x} = \dfrac{\delta \varphi}{\delta y} = 0$, all other values are found relative to this datum.

The boundary conditions are (i) shear stresses equal zero on all boundaries (ii) normal stresses are zero except on the supports. Boundary values are determined as follows : *

For boundary AB the value of φ is zero and for BC applying the equation $\sigma_x = \dfrac{\delta \varphi^2}{\delta y^2} - \rho g \cdot y = 0$ and integrating twice with respect to y then $\varphi = \frac{1}{6} \cdot \rho g \cdot y^3$. Again on this boundary $T_{xy} = -\dfrac{\delta^2 \varphi}{\delta x \delta y} = 0$ or $\dfrac{\delta \varphi}{\delta x} + C_1 = 0$, but at $y = 0$ (from boundary AB) $\dfrac{\delta \varphi}{\delta x} = 0$, hence C_1 should be zero. Now considering the boundary CD_1 where a uniform pressure 'p' is assumed to be = reaction $R/(\alpha \cdot S)$ it is clear that $-p = \dfrac{\delta^2 \varphi}{\delta x^2} - \rho g \cdot y$. Integrating twice with respect to x we get the equation $\dfrac{px^2}{2} + \varphi - \rho g \cdot \dfrac{Hx^2}{2} + C_2 x + C_3 = 0$. From boundary BC : $\dfrac{\delta \varphi}{\delta x} = 0$ and $\varphi = \rho g \cdot \dfrac{H^3}{6}$ at point C, then C_2 and C_3 can be evaluated giving $\varphi = -\dfrac{p}{2}(S - x)^2 + \frac{1}{2} \rho g \cdot H \left[(S - x)^2 + \dfrac{H^2}{3} \right]$.

Regarding the shear stress we have $T_{xy} = -\dfrac{\delta^2 \varphi}{\delta x \delta y} = 0$ and it is evident that for all the points $\dfrac{\delta \varphi}{\delta y} = -\rho g \cdot \dfrac{H^2}{2}$.

Between D_1 and D_2 the same procedure is applied and using the equation of φ for CD_1 we get

$$\varphi = \frac{1}{2} \left\{ R \left[\alpha S - 2(S - x) \right] + \rho g \cdot H \left[(S - x)^2 + \frac{H^2}{3} \right] \right\}$$

and $\dfrac{\delta \varphi}{\delta y} = -\rho g \cdot \dfrac{H^2}{2}$

The effective density of the material used in the experiments was 110 gm/cm³. The results obtained depend on the spread of the reaction, and to examine the effect of this on the stresses in the beams two cases have been examined (a) with a concentrated reaction force and (b) assuming the reaction (R) to be spread over a length of $\alpha S = 0 \cdot 185$ c.m., i.e. equal to $0 \cdot 074$ of half the span (S) at each support. The boundary values for each of these cases are indicated in Fig. 3. With regard to the external values at points 16′ and 20′ it is apparent that as $\dfrac{\delta \varphi}{\delta y} = -1375$ then $\frac{1}{2}(\varphi_{16} - \varphi_{16}') = -1375$ or $\varphi_{16}' = \varphi_{16} + 2750$ and similarly at the other points on this line.

Appendix III

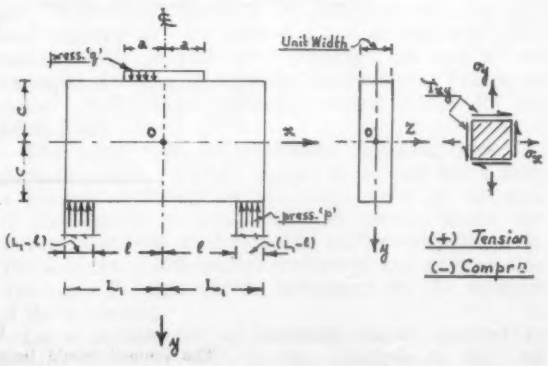


Fig. 16

*An alternative method has been described by J. H. A. Brahtz cf. *Proc. A.S.C.E.*, Vol. 61, 1935.

APPENDIX III

Referring to fig. 16 and denoting

$$\alpha = m\pi/L_1 \quad , \quad \beta = \alpha c \quad , \quad \alpha x = \xi \quad , \quad \alpha y = \eta$$

$$\beta - \eta = u \quad , \quad \beta + \eta = v \quad \text{and}$$

$$K = \frac{L_1^2}{4c^3}\left\{ p\left[\frac{\ell}{L_1} - \left(\frac{\ell}{L_1}\right)^3\right] + q\left[\frac{a}{L_1} - \left(\frac{a}{L_1}\right)^3\right]\right\}$$

$$\therefore \quad \sigma_x = \left[K y + \sum_{m=1}^{\infty} \frac{h_m \cos\xi}{\sinh^2 2\beta - 4\beta^2} \times \right.$$

$$\left\{ \sinh 2\beta\,(u\cosh v - \sinh v) - 2\beta(v\cosh u - \sinh u)\right\}$$

$$+ \sum_{m=1}^{\infty} \frac{a_m \cos\xi}{\sinh^2 2\beta - 4\beta^2} \left\{ \sinh 2\beta\,(v\cosh u - \sinh u)\right.$$

$$\left.\left. - 2\beta\,(u\cosh v - \sinh v)\right\}\right]$$

$$\sigma_y = \left[\frac{1}{2L_1}\left\{ q.a + p(L_1 - \ell)\right\} - fg \times y - \right.$$

$$\sum_{m=1}^{\infty} \frac{h_m \cos\xi}{\sinh^2 2\beta - 4\beta^2}\left\{ u\sinh 2\beta\cosh v - 2\beta v\cosh u + \right.$$

$$\left.\sinh 2\beta \sinh v - 2\beta \sinh u\right\} - \sum_{m=1}^{\infty} \frac{a_m \cos\xi}{\sinh^2 2\beta - 4\beta^2} \times$$

$$\left.\left\{ v\sinh 2\beta\cosh u - 2\beta u\cosh v + \sinh 2\beta \sinh u - 2\beta \sinh v\right\}\right]$$

$$\tau_{xy} = \sum_{m=1}^{\infty} \frac{h_m \sin\xi}{\sinh^2 2\beta - 4\beta^2}\left\{ u\sinh 2\beta \sinh v + 2\beta v \sinh u\right\} -$$

$$\sum_{m=1}^{\infty} \frac{a_m \sin\xi}{\sinh^2 2\beta - 4\beta^2}\left\{ v\sinh 2\beta \sinh u + 2\beta u \sinh v\right\}$$

where :

$$a_m = \frac{2q}{m\pi}\sin\alpha a \quad , \quad h_m = -\frac{2p}{m\pi}\sin\alpha\ell$$

& fg = Density of material

In a beam of infinite depth $\sigma_x = \sum_{m=1}^{\infty} h_m \cos\xi\,(u-1)\,e^{-u}$

(e = base of Naperian log.)

Discussion

The Council would be glad to consider the publication of correspondence in connexion with the above paper. Communications on this subject intended for publication should be forwarded to reach the Institution by September 30th, 1961.

A Review of Information on the Settlement of Structures

by S. Rosenak, B.Sc., A.M.I.Struct.E., A.M.I.C.E.

Synopsis

The paper reviews the methods of predicting the settlement of structures on spread footings and piled foundations. It reports settlement observations on structures and model tests. Reference is made to the stiffness of the structure and its effect on settlement. The limits of permissible settlement to prevent cracking or damage are discussed.

Notation

B = Breadth of footing.
c = Apparent cohesion.
c_v = Coefficient of consolidation.
c_{kd} = Dutch cone penetration resistance (in kg./sq. cm.).
C = Constant of compressibility.
d = Diameter of pile.
e = Voids ratio.
E = Young's modulus.
E_h = Elastic heave.
$E'I_B$ = Flexural rigidity of structure.
E'_sB^3 = Flexural rigidity of soil.
F = Factor of safety.
H = Thickness of soil layer.
I_ρ = Influence coefficient depending on the shape of the loaded area and the depth of soil layer.
k = Modulus of subgrade reaction.
k_s = Modulus of subgrade reaction of a footing B ft wide.
k_{s1} = Modulus of subgrade reaction of a footing 1 ft wide.
K = Permeability (Darcy's coefficient).
K_r = Ratio of stiffness of structure to that of soil.
L = Length of footing.
m_v = Coefficient of compressibility.
N = Standard spoon penetration resistance.
p_o = Vertical stress before application of load.
Δp = Stress increase.
p_z = Vertical effective stress at depth z.
q = Pressure intensity.
q_a = Allowable bearing pressure.
qf = Ultimate bearing capacity.
q' = Effective pressure.
t = Time.
T_v = Time factor.
U = Degree of consolidation.
u = Pore water pressure.
y = Vertical settlement of a footing B ft wide.
y_1 = Vertical settlement of a footing 1 ft wide.
z = Depth below surface.
γ_w = Unit weight of water.
Δ = Differential settlement.
Δl = Ratio of central deflexion to span.
δl = Angular distortion.
ρ = Settlement.
ρ_c = Consolidation settlement.
ρ_i = Immediate or elastic settlement.
ρ_{oed} = 'Oedometer' settlement.
ρ_t = Consolidation settlement at time t.
μ = Consolidation test correction factor.
ν = Poisson's ratio.

Introduction

It is the usual practice today to consider in some detail at the design stage the probable settlement of any important structure. Considerations of settlement are important for a number of reasons :

Buildings are becoming taller, with consequently higher foundation loads and pressures.

On congested sites new buildings erected close to existing structures may cause increased differential settlement and possible damage in the existing structures.

Industrial structures, warehouses and heavy structures, such as atomic power stations and oil refinery installations, are being built on estuarine soils which may consist of deep layers of compressible material.

Because of the shortage of building land sites which would normally be considered unsuitable may have to be utilised for building purposes.

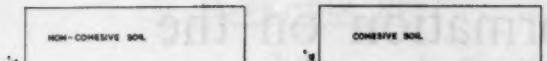
Volume changes in the soil may be caused by climate, vegetation and industrial processes. The resulting soil movement may affect especially lightly loaded structures.

Bearing Capacity and Settlement

The bearing capacity of a soil is related to its shearing strength. If the soil below a footing is overstressed there will be a sudden rupture of the soil mass with shearing between the soil particles and slipping along a curved plane of rupture. The footing will tilt and settle rapidly. A well known example of a bearing capacity failure is the Transcona grain silo [1] which suddenly tilted while it was being filled.

The usual type of settlement problem, however, does not involve a shear failure of the soil but simply a gradual reduction in the voids ratio of the soil. If the change in voids ratio is uneven under the structure uneven or differential settlement will result. The amount of differential settlement and any possible structural damage will be influenced by the stiffness of the structure.

Large settlements of buildings have occurred in Mexico City.[2] A well known example is the Old School of Engineering which has settled several feet, the settlement contour being bowl shaped. The leaning Tower of Pisa represents an extreme case of differential



Figs. 1 and 2—Allowable bearing pressures of footings on non-cohesive and cohesive soils

settlement. This tower, which is 150 ft high, is out of plumb by 14 ft, i.e., by about 10 per cent of its height. Its walls are, however, sufficiently massive to prevent collapse of the structure.

The criteria which determine the allowable bearing pressure below a footing may be either the ultimate bearing capacity of the soil, divided by a suitable factor of safety, or the permissible settlement. In general the settlement of shallow footings on both cohesive and non-cohesive soils is proportional to the width of the footing. For shallow footings on non-cohesive soils the ultimate bearing capacity increases with the width of the footing, whereas on cohesive soils it is independent of the width of the footing. For both types of soil two intersecting curves can be drawn, as shown diagrammatically in Figs. 1 and 2, to indicate whether bearing capacity or settlement considerations will determine the allowable bearing pressure.

For non-cohesive soils, where the bearing capacity increases rapidly with increasing width of the footing, settlement considerations will usually govern the allowable bearing pressure. In the case of cohesive soils settlement is a long term process, so that initially the allowable bearing pressure may be limited by the ultimate bearing capacity of the soil.

The Settlement of Shallow Foundations

(a) *Elastic or Immediate Settlement*

The vertical elastic settlement below a corner of a flexible rectangular loaded area on a homogeneous soil is given by the following equation developed by Schleicher [3] :

$$\rho i = q \cdot B \cdot \frac{(1 - \nu^2)}{E} \cdot I\rho$$

Poisson's ratio for soils is usually taken as $\frac{1}{2}$, which reduces the above equation to :

$$\rho i = \frac{3}{4} q \cdot \frac{B}{E} \cdot I\rho$$

This equation applies to a soil of infinite depth. It has been developed further by Steinbrenner [4] to deal with soil layers of finite depth. The constant $I\rho$ is replaced by another constant depending on the geometry of the problem.

In order to determine the theoretical settlement at a particular point under a large footing, the footing is divided into a number of rectangles and the influence factor $I\rho$ is determined for each separate rectangle. It is also possible to replace the loaded area by a series of point loads and to determine the stresses (and hence the strains or settlements) at different depths by means of Boussinesq's equations. The spacing of the point loads should not exceed $z/3$, where 'z' is the depth below the loaded area under consideration.

From the above equations it is seen that the settlement increases with the width of the loaded area. This applies to both elastic and long-term consolidation settlements.

Table A
Tangent Modulus of Sand

Confining pressure (tons/sq. ft.)	Tangent Modulus (tons/sq. ft.)	
	Loose sand	Dense sand
5	500	3,500
10	1,000	4,500
15	1,500	5,500
20	2,000	6,000

[After Terzaghi and Peck[5]]

Although soils in general do not obey Hooke's Law and have volume changes which are only partially reversible, it is possible to assign an E-value to soils. This may be the 'tangent' or 'secant' modulus applicable to a particular loading condition. In Table A some experimental values are given of the tangent modulus of sands [5]. The influence of the confining pressure on the value of 'E' is apparent from this Table. As the lateral pressure on a deep layer of sand increases with depth, its E-value will also increase. The settlement of a footing on sand or other non-cohesive soil may therefore be expected to increase not simply with the width of the loaded area but in proportion to the B/E-ratio.

In connection with soils the term 'modulus of subgrade reaction' is often used instead of the 'modulus of elasticity.' The modulus of subgrade reaction is defined as :

$$k = \frac{q}{y} = \text{constant.}$$

Experiments[6] have shown that for footings on sand of different widths but with the same unit pressures the settlements are related by the equation :

$$\frac{y}{y_1} = \left(\frac{2B}{B+1}\right)^2$$

Similarly the moduli of subgrade reaction 'k_s' and 'k_{s1}' for footings of width B ft and 1 ft respectively are related by :

$$k_s = k_{s1}\left(\frac{B+1}{2B}\right)^2$$

i.e., the settlement of a wide footing on sand may be up to four times the settlement of a 1 ft wide footing.

According to Terzaghi[6] the settlement of a 1 ft square footing on sand is approximately the same as that of a long footing 1 ft wide, so that 'k_{s1}' can be determined experimentally by loading a 1 ft square plate. He suggests the following values for 'k_{s1}' (Table B):

Table B
Values ks_1 (in tons/cu. ft.) for a 1ft. square plate or a 1ft. wide beam.

Relative density of sand	Loose	Medium	Dense
Dry or moist sand, range	20-60	60-300	300-1,000
Dry or moist sand, proposed values	40	130	500
Submerged sand, proposed values	25	80	300

[After Terzaghi[6]]

On clays the elastic settlement increases approximately in direct proportion to the width of the loaded area. For practical purposes they behave essentially as materials with a constant value of '*E*'.

It is comparatively easy to obtain undisturbed samples of clay soils from small diameter boreholes. The *E*-value can be determined experimentally by means of the triaxial compression test. In the case of granular soils loading tests to determine '*E*' or '*k*' can only be performed near the surface. It is difficult to obtain satisfactory undisturbed samples at any appreciable depth. It is therefore usual to assess the density of granular soils, and hence the bearing capacity and settlement, by means of penetration tests. In the dynamic "standard penetration test" a standard sampling spoon is driven into the soil and the number of blows for a penetration of 12 in. ('*N*') is measured. The *N*-value gives an indication of the relative density of the soil. Terzaghi and Peck[5] have suggested allowable bearing pressures depending on the *N*-value and the width *B* of the footing, to limit the maximum settlement of individual footings to 1 in. and the differential settlement to $\frac{3}{4}$ in. For footings between 5 and 15 ft the allowable bearing pressure in tons/sq. ft is approximately equal to $N/10$.

Recent investigations[7] have shown that the *N*-value increases with overburden pressure even if the relative density remains constant. A submerged sand would be expected to have a lower *N*-value than a dry or moist sand at the same relative density. A coarse sand compacted to a relative density of 60 per cent for instance, would have the following approximate *N*-values shown in Table C :

Table C
Standard Penetration Resistance "N" of a coarse sand at a relative density of 60%

State of sand	Overburden pressure (lb/sq. in.)	N-value
Dry	0	6
	20	17
	40	24
Saturated	0	4
	20	11
	40	18
Terzaghi & Peck value for air dry sand		26

[After Gibbs and Holtz']

It may therefore be assumed that the standard penetration test makes some automatic correction for the effects of overburden pressure and submergence. The bearing pressures proposed by Terzaghi and Peck may be considered to be on the safe side for shallow footings.

The Dutch static cone penetrometer is frequently used in deep layers of sand. A cone is pushed into the soil and the cone penetration resistance is measured. The constant of compressibility *C* of the sand is determined from the relation[8] :

$$C = \frac{3}{2} \cdot \frac{C_{kd}}{pz}$$

and the settlement is calculated from the relation :

$$p = \sum_{H1}^{H2} \frac{H}{C} \cdot \times 2{\cdot}3 \,(log\,(p_o + \Delta p) - log\,p_o)$$

This method has been used to estimate the settlement of a number of bridges in Belgium founded on deep layers of sand.

(b) *Consolidation Settlement*

Structures founded on cohesive soils are subject to slow consolidation settlements. This type of settlement may continue for many years after the completion of the structure, and it is obviously important to be able to predict its magnitude. Cohesive soils are usually fully saturated, and a reduction in the volume of the soil, caused by the application of a load for instance, can only take place by forcing some of the water out of the soil. Because of their low permeability this process takes place very slowly. In Table D some typical values of the permeability of soils are quoted, from which it will be seen that the passage of water through cohesive soils is so slow that they may be virtually impervious.

Table D
Permeability of soils

Soil type	Permeability (cm./sec.)
Clean gravel	100 — 1
Clean sands	1 — 10^{-3}
Very fine sands and silts	10^{-3} — 10^{-4}
Clays	10^{-4} — 10^{-9}

When a load is placed on a cohesive soil this load is at first carried entirely by the water in the pore space, and a pore water pressure is developed equal to the applied pressure. As the water gradually flows out of the soil, its volume compresses and causes the applied load to settle. At the same time the 'pore water' or 'neutral' pressure decreases and the 'intergranular' or 'effective' pressure increases, until eventually the applied load is carried entirely by the soil structure. The pore water pressure will then have dropped to zero and the flow of water will cease, i.e., no further reduction in the volume of the soil will take place and the settlement will be completed.

For a saturated soil the relationship between total and effective pressure is given by the well known equation :

$$q = q' + u,$$

$$\text{or } q' = q - u.$$

For saturated soils there is usually a definite relationship between the voids ratio and the effective pressure. All settlement computations are based on a determination of the change in the applied pressure and the corresponding change in the voids ratio. This relationship is found experimentally in the oedometer apparatus by using undisturbed soil samples.

The following coefficients are used for settlement computations :

coefficient of compressibility

$$m_v = \frac{-de}{dp\,(1+e)} \text{ (sq. ft/ton)},$$

coefficient of consolidation

$$c_v = \frac{k}{\gamma_w \cdot m_v} \text{ (sq. ft/year)},$$

time factor

$$T_v = \frac{c_v \cdot t}{H^2},$$

degree of consolidation

$$U = \frac{\rho t}{\rho f}.$$

It may be of interest to quote typical values of some of these coefficients as well as E-values for London clay[9]:

m_v : 0·01 to 0·003 sq. ft/ton,
c_v : 3 to 8 sq. ft/year,
E : 250 to 350 tons/sq. ft.,
E_h : 500 to 1,100 tons/sq. ft.

The elastic heave of London clay was measured recently on a particular site in London by Serota and Jennings[10] in connexion with the demolition of a building. Their measured value of the elastic heave was: $E_h = 1{,}200$ tons/sq. ft. There is an approximate relationship between 'm_v' and 'E' and the cohesion 'c' of London clay as follows[9]:

$$\frac{1}{m_v \cdot c} = 100, \text{ and } E = 140\,c,$$

where E, m_v and c are measured in ft. ton-units. Similar relationships can be found for other types of clay. In a more recent paper[11] it has been suggested that for London clay the relationship $\frac{1}{m_v \cdot c} = 100$ applies to the fissured clay and that $\frac{1}{m_v \cdot c} = 75$ applies to the clay in the less fissured state. Under repeated loading cycles an approximate relationship was found between 'm_v' and 'E' for London clay as follows:

$$E = \frac{3 \cdot 9}{m_v}.$$

For ordinary spread footings it is usual to estimate the theoretical elastic and consolidation settlements at the centre of the footing, assuming a flexible load. Corrections are then made for the rigidity of the footing and its depth below the surface. The depth correction can be applied on the basis of graphs prepared by Fox[12].

A general correction has also to be made to the calculated settlement on the basis of the oedometer test, ρ_{oed}. It has been found, for instance, that the actual settlements of footings on overconsolidated clays are smaller than those expected on the basis of the oedometer tests. A method of estimating the correction factor 'μ' to be applied to settlement computations was developed by Skempton and Bjerrum[13]. The value of 'μ' depends largely on the geological history of the clay. For very sensitive clays 'μ' may exceed 1·0, for normally consolidated clays 'μ' is slightly less than 1·0, while for very heavily overconsolidated clays 'μ' may be as low as 0·25. For London clay 'μ' is usually taken as 0·5.

If the theoretical consolidation settlement is

$$\rho_{oed} = \sum_{z1}^{z2} m_v \cdot \Delta p \cdot z$$

the actual consolidation settlement will be

$$\rho_c = \mu \cdot \rho_{oed}$$

and the total settlement at any particular time 't' will be

$$\rho_t = \rho_1 + U \cdot \mu \cdot \rho_{oed}$$
$$= \rho_1 + U \cdot \rho_c.$$

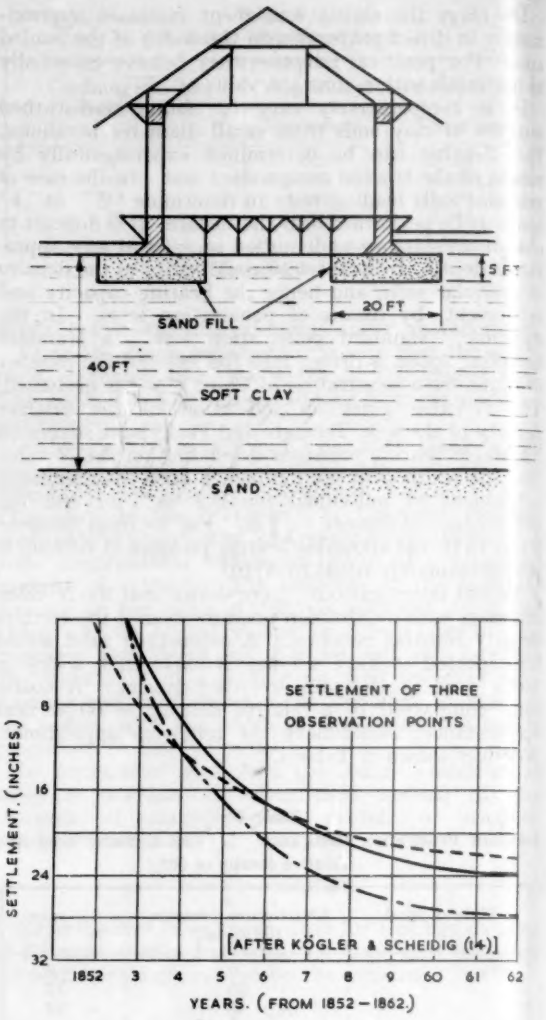


Fig. 3—Settlement of freight shed on clay

(*c*) *Settlement Observations on Structures*

Koegler and Scheidig[14] describe several cases of ancient public buildings and cathedrals in North Germany which have settled as much as 6 ft. These buildings are usually massive masonry structures resting on 30 to 50 ft of compressible peaty and clayey soils. Such large settlements would not be permissible in modern framed structures, and some type of semi-buoyant foundation or piled foundation would be required to reduce the settlement. The settlement of a freight shed is described which had settled 24 in. between 1852 and 1862. The site conditions are shown diagrammatically in Fig. 3. It may be mentioned that the provision of a large sand blanket under the footings was an unnecessary expense as it did not prevent the consolidation of the 40 ft. deep layer of clay.

Observations have been made of the settlement of bridge foundations founded on deep layers of sand in Belgium[8]. Some of the bridges were continuous structures which are more sensitive to differential settlement than simply supported spans. The maximum and minimum values of the constant of compressibility 'C' were found from the Dutch cone

penetration tests. A weighted value of 'C' was calculated for each layer, and this value was used for settlement computations. The sizes of the bridge abutments or piers varied from about 3 m. by 14 m. to 5 m. by 52 m. The pressures under the footings were of the order of 2 to 3 tons/sq. ft, and the measured settlements varied between 1 and 5 cm. The maximum measured differential settlement between parts of the footings were found to range between 15 and 45 per cent of the maximum settlement. A comparison of the calculated settlements based on a weighted average value of 'C' and the actual settlements showed that the ratio $\frac{\text{calculated settlement}}{\text{measured settlement}}$ was higher than 1 but always less than 2. The calculated settlement gave therefore an upper safe limit which was never exceeded. In view of the heterogeneous nature of the sand layers the agreement between calculated and observed settlement was considered satisfactory.

Many settlement observations have been made on structures founded on clay soils. Careful records have been made of a number of buildings in America on Chicago clay. Settlement observations have been made for sixty years on the Chicago Auditorium Tower which has settled 24 in. since its erection in 1887. The Building Research Station has made observations on buildings and bridges in this country. The results of these observations have been analysed in the previously mentioned paper by Skempton and Bjerrum [13]. Their proposed method should make it possible to obtain a reasonable estimate of the probable settlement of a structure on clay. In estimating loads for settlement computations the dead load and only a portion of the live load have to be considered. In Fig. 4 the observed and estimated settlement of an oil tank [15] is shown. It is of interest to note the magnitude of the immediate or elastic movement of the structure during a loading and unloading cycle.

Table E
Settlement Ratios for Foundations on Sand

Width of Foundation	$\frac{\text{Foundation settlement}}{\text{Settlement of test pile or plate}} = \frac{\rho B}{\rho 1}$	
B ft.	Spread footing	Driven piles in sand
5	2·4	3·3
10	3·2	5
15	3·5	6·5
20	3·6	7·5
25	3·7	8·3
30	3·75	9
40	3·8	10

[After Skempton[16]]

Settlement of Piled Foundations

The settlement of a pile group is usually greater than that of an individual pile at the same load per pile. One may imagine the pile group to be replaced by a block of soil composed of the soil and piles of the same overall dimensions as the pile group. The settlement of the block would be expected to increase with increasing width of the pile group, in a manner similar to that of a shallow footing. This would apply to both end-bearing piles in granular soils and friction piles in cohesive soils. In the case of piles in granular soils it is assumed that the superimposed load is transferred to the ends of the piles, and the

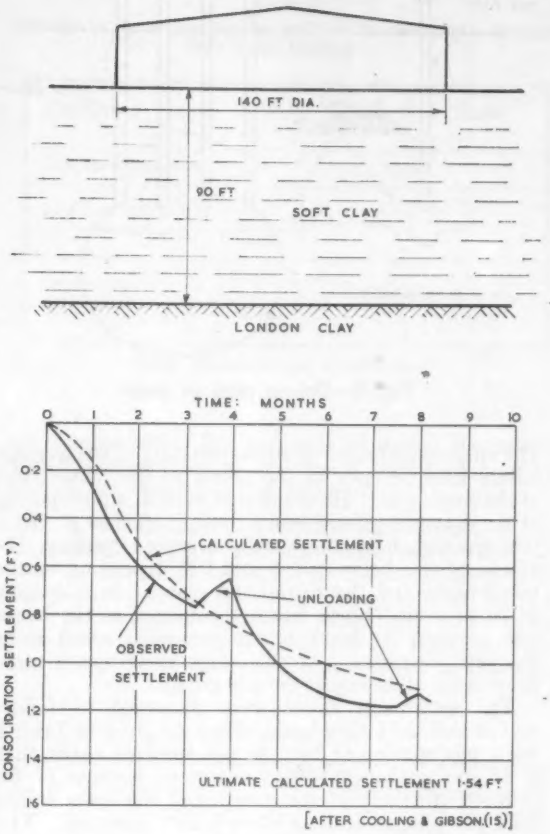


Fig. 4—Observed and calculated consolidation settlement of centre of oil tank

stress distribution and settlement are determined accordingly. In cohesive soils, where a large part of the load is carried by skin friction, the superimposed load is assumed to act at a depth of $\frac{2}{3}$ of the pile length for stress distribution and settlement calculations.

The ratio between the settlement of the pile group and that of an individual pile is usually termed the "settlement ratio". For shallow footings on sand the settlement ratio (ratio of settlement of a footing of width B ft to that of a footing 1 ft wide) reaches a limiting value of 4. A limited number of test records is available of driven piles in sand. They tend to show that the settlement ratio may be as high as 12. Skempton [16] has suggested a tentative relationship on the basis of these tests. His values and those of shallow footings on sand are compared in Table E.

If, for instance, the settlement of a pile group 20 ft wide is to be limited to 2 in., the settlement of an individual test pile should not exceed $2/7{\cdot}5 = 0{\cdot}265$ in. The load on the test pile at this settlement would be the working load per pile in the pile group.

Skempton explains the large settlement ratio of a pile group in sand by the fact that a narrow band of soil below the toe is compacted during driving. The bulb of pressure of an individual pile lies in this band, whereas the bulb of pressure of the pile group extends to a greater depth, as shown in Fig. 5. The compacted zone is absent in shallow spread footings.

Whitaker [17] has performed model tests on friction piles on clay. He examined square groups of piles and observed their efficiencies and settlement ratios.

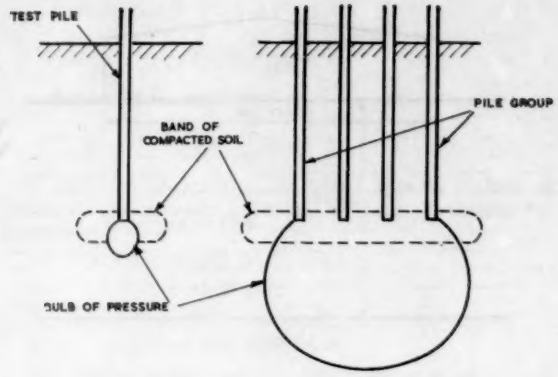


Fig. 5—Driven piles in sand

The efficiency of a pile group is the ratio of the average failure load per pile in the group to the failure load of the single pile. He found that even at a pile spacing of six diameters there was a drop in efficiency. At a pile spacing of $3d$, the usually accepted spacing, the efficiency was between 0·8 and 0·7 depending on the pile lengths and the size of the group. At a spacing of $2d$ to $2{\cdot}5d$ " block failure " occurred in the larger pile groups. At block failure the soil sheared along the pile perimeter. At this stage there was a rapid drop in the efficiency of the pile group.

The settlement ratios were measured at failure and at half the failing load. They are given in Table F for a pile spacing of $3d$. At pile spacings closer than $2{\cdot}5d$ the settlement ratios decreased because of the reduced efficiency of the group, i.e., the lower failing load per pile, as soon as block failure occurred. At a spacing of $1{\cdot}5d$, when block failure took place, the settlement ratios at half the failing load were about 7 for piles $24d$ long and about 10 for piles $48d$ long.

It was also observed in the tests that under " working loads " the outer piles took more load than the inner piles. As failure approached the load on the outer piles remained stationary while the load on the inner piles increased. The soil surrounding the outer piles appeared to have reached a plastic state while it was still elastic near the centre of the pile group.

In all the tests the pile cap was rigid and the investigations were concerned with " immediate " settlements. The important point to note is the magnitude of the settlement ratios. This has to be kept in mind in assessing the behaviour of a pile group on the basis of loading tests on single piles.

Table F

Settlement Ratios of square groups of test piles in clay at a pile spacing of $3d$.

Pile length	Pile group X²	Settlement ratio at	
		failing load	½ failing load
$24d$	3²	3	2
	5²	6	4
	7²	13	6
	9²	25	10
$48d$	3²	4	4
	5²	9	7
	7²	22	15
	9²	36	22

Note : Pile group X² = X piles along side of square.

[After Whitaker[17]]

A pile loading test and an observation of the settlement of the piled structure, a boiler house and a coal bunker, has been reported by Lumpert[18]. Driven cast in situ piles 16 m. long were used. The piles were driven through a lake marl into a sandy loam. The soil was therefore slightly cohesive with probably a high coefficient of consolidation. In the pile test a load of 60 tons was left on the pile for six days, during which time the settlement had reached about 7 mm. At that stage the settlement curve was flattening out rapidly. Three years after the completion of the structure its average settlement under an average pile load of 40 tons, including the live load, was 80 mm., i.e., it was eleven times as great as the settlement of the test pile under a greater load. The settlement is now virtually completed, but it fluctuates slightly with changes in the live load.

In connexion with settlement computations of friction piles reference may be made to the measured settlement of a bridge abutment on friction piles in Denmark.[19] Timber piles about 16 m. long were used. They were driven into a soft post-gracial clay, and the settlement after 15 years was about 80 cm. In this clay a large part of the settlement was due to secondary consolidation. Settlement computations were made on the assumption that the load was acting either at the pile ends or at a depth of ⅔ of the pile length. Both methods were found to underestimate the actually observed settlement, but the second assumption gave a better estimate of the measured settlement.

Limits of Permissible Settlement

It is important to be able to assess the effects of the total or differential settlement of a structure on its serviceability. In a warehouse floor on which goods are to be stacked mechanically even a small amount of differential settlement may be objectionable. Storage tanks on soft soils may settle provided they do not tilt because of shear failure of the soil. In a rigid box-like structure the total settlement may be unimportant since the differential settlement is expected to be small. There may be architectural or aesthetic considerations which limit the allowable settlement or tilting of a structure, even though such movement would not endanger the stability of the structure.

Meyerhof[20] has investigated the settlement behaviour of a structure by considering the structure and the underlying soil as a combined structural unit. He has made a theoretical analysis which is applicable mainly to structures on cohesive soils. It is assumed that the combined unit behaves elastically, since under working loads the soil is only stressed to about one third of its ultimate bearing capacity. He uses an effective modulus of deformation of the soil " E'_s " which includes the elastic, plastic and consolidation characteristics of the soil. The flexural rigidity of the structure is given by $E'I_B$. The distribution of the contact pressure on the base depends on the relative stiffness K_r of the foundation ($E'I_B$ per unit width or length) to that of the soil (E'_sB^3).

Meyerhof has computed the maximum total and differential settlement for structures of different shapes and relative stiffnesses " K_r." His computations show that the total settlement increases with the length/width ratio L/B of the foundation, but that the total settlement is not greatly influenced by the relative stiffness " K_r." The differential settlement, however, decreases rapidly as the relative stiffness " K_r " increases. For $Kr = 0$, the ratio of

differential to total settlement is 0·5 for a long foundation and about 0·35 for a square foundation. For $K_f = 0{\cdot}5$ the ratio of differential to total settlement is about 0·1.

An approximate assessment has to be made of the value of $E'I_B$. It may be taken as the sum of $E'_B I_B$, the flexural rigidity of the superstructure, and $E'_F I_F$, the flexural rigidity of the foundation. Where a building rests on individual footings the flexural rigidity of the whole foundation is usually small compared with that of the superstructure. The superstructure may be an open multi-storey frame or a solid frame consisting of a continuous wall cladding. In the case of the open frame,

$$E'I_B = \Sigma\, E'I'_b,$$

where $\Sigma\, E'I'_b$ is the sum of the effective flexural rigidities of each floor beam. If the frame is a solid load bearing wall, then, neglecting any openings,

$$E'I_B = \frac{E'bh^3}{12},$$

where "b" and "h" are the average thickness and height of the wall respectively.

The theoretical settlement stresses induced in structural members may be very large, but the actual stresses are probably much smaller because of some form of composite action of the structure and because of creep effects. It is therefore difficult to predict the amount of settlement or distortion which would cause damage to the structure, and it is necessary to resort to tests and to observe structures which have settled. Meyerhof [20] reports some observations and tests carried out by the Building Research Station. In quick loading tests on brickwalls supported on concrete beams cracking of the brickwalls started at a ratio of central deflexion to span Δ/l of 1/5,000. Observations of brickwalls of buildings indicated that under field conditions cracking was likely at Δ/l = 1/1,000. In racking tests on cased steel frames with brick infilling panels cracking of the panels was observed at an angular distortion of δ/l of 1/300 to 1/400. Cracking of the encasement of a similar open frame started at a δ/l of about 1/100. Meyerhof suggested limiting values of angular distortion of δ/l of 1/300 for an open frame and 1/1,000 for a frame with wall panels between the columns.

Skempton and MacDonald[21] have made a very thorough study of case records of buildings and structures which have been damaged on account of settlement. They investigated the total settlement ρ, the angular distortion δ/l, and the differential settlement Δ of panelled and framed buildings. They found that at an angular distortion greater than 1/300 panel damage was likely, but structural damage to the frame was only likely to occur at an angular distortion of more than 1/150. Taking $\delta/l = 1/300$ as the criterion for panel damage, they found an approximate correlation between δ/l and the differential settlement Δ. There was a marked difference between buildings on clay and buildings on sand. For clay the correlation was

$$\Delta = 550 \cdot \delta/l,$$

and for sand it was

$$\Delta = 350 \cdot \delta/l.$$

In Table G, which is taken from the paper by Skempton and MacDonald, the results of their findings relating to differential and total settlement are summarised. For purposes of design Skempton and MacDonald suggest factors of safety of 1·25 to 1·5, on the values given in Table G.

Table G

Damage limits for load-bearing walls or for panels in traditional-type frame buildings.

Criterion		Isolated foundations	Rafts
Angular distortion		1/300	
Greatest differential settlement	Clays	1¾ in.	
	Sands	1¼ in.	
Maximum settlement	Clays	3 in.	3 to 5 in.
	Sands	2 in.	2 to 3 in.

[After Skempton and MacDonald[21]]

It is of interest to compare the damage limits found by Skempton and MacDonald with those of other investigators. An analysis of settlement records has been made in Russia[22]. On the basis of this analysis values of total and differential settlement have been incorporated in the latest Russian Building Code. Some of these values are given in Table H.

Table H

Type of Structure	Differential Settlement	
	Sub soil	
	Sand and hard clay	Clay in plastic state
Slope of craneway tracks	0·003	0·003
Differential settlement between columns of buildings		
(a) Steel & r.c. framed structures	0·002L	0·002L
(b) End rows of columns with brick cladding.	0·007L	0·001L
Relative deflexion of plain brick walls		
(a) Multi-storey buildings, $L/h \leq 3$	0·0003	0·0004
$L/h \geq 5$	0·0005	0·0007
(b) One storey mills	0·001	0·001
Pitch of solid or ring shaped foundations of high rigid structures (smoke stacks, water towers, silos)	0·004	0·004
	Average Settlement	
Buildings with plain brick walls on continuous and separate foundations		
$L/h \geq 2{\cdot}5$	3 in.	
$L/h \leq 1{\cdot}5$	4 in.	
Buildings with brick walls, reinforced with r.c. or reinforced brick belts.	6 in.	
Framed buildings	4 in.	
Solid r.c. foundations of smoke stacks, water towers, silos.	12 in.	

Extracts of the Building Code of the U.S.S.R., 1955[22]

In general the differential settlement limits are rather less but the total permissible settlement limits are greater than the values suggested by Skempton and MacDonald.

Conclusions

It is possible to predict, on the basis of soil tests, with reasonable accuracy the settlement of structures. The overall settlement behaviour of a structure depends on the relative stiffness of the structure and the underlying soil. Settlement observations on structures have helped to improve methods of settlement computations.

From a study of case records it has been possible to fix approximate limiting values of settlement for certain types of structures to prevent damage or cracking. It should be mentioned that soil movement can also be caused by natural volume changes in the soil or by industrial operations. Where the magnitude of the soil movement is known, its effect on a structure may be assessed from a knowledge of the damage limits due to settlement or distortion of the particular type of structure.

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Discussion

The Council would be glad to consider the publication of correspondence in connexion with the above paper. Communications on this subject intended for publication should be forwarded to reach the Institution by September 30th, 1961.

Book Review

Methods and Techniques in Geophysics, Vol. 1, Edited by S. K. Runcorn. (New York : Interscience Publishers Inc., London : Interscience Publishers Ltd., 1960). 9 in. × 6 in., 374 plus ix pp., 72s. ($10.00).

This book, which has been compiled to set forth the modern advances in physical techniques in geophysics, contains ten papers contributed by experts working in Great Britain, Canada and the United States.

The first paper, by E. C. Bullard of the University of Cambridge, deals with the measurement of temperature gradient on land, in boreholes and in mines and tunnels, and at sea. This is followed by a paper on the measurement of heat flow over land, including temperature gradient and thermal conductivity and calculation of flux, by A. D. Misener and A. E. Beck of the University of Western Ontario. Borehole surveying is dealt with by T. F. Gaskell and P. Threadgold of the British Petroleum Company, London, and the measurement of the geomagnetic elements by K. Whitham of the Dominion Observatory, Ottawa. D. W. Collinson and K. M. Creer of King's College, University of Durham, contribute a paper on measurements in palaeomagnetism and J. C. Harrison of the University of California deals with the measurement of gravity at sea. In a paper on the detection of earth movements, P. L. Willmore of the Dominion Observatory, Ottawa, deals with the representation of seismograph response, the theory of the pendulum seismographs and the measuring of stress and strain. The measurement of electric currents flowing through the earth, the nature of earth-current records and electromagnetic induction in the earth are described in a paper by G. D. Garland of the University of Alberta, Canada, and D. S. Hughes of the University of Texas deals with the properties of rocks under high pressure and temperature. The volume concludes with a paper on latitude and longitude and the secular motion of the pole by W. Markowitz of the United States Naval Observatory, Washington.

A comprehensive bibliography is added to each paper, and the volume is attractively produced and well illustrated.

Bolted Beam to Column Connexions

by A. N. Sherbourne, Ph.D., M.A., A.M.I.Struct.E.

Synopsis

The paper describes an experimental investigation into the structural behaviour of a particular type of bolted beam-column connexion, in which the connexion consists of a plate welded to the end of a beam and fastened to the flanges of the column by pretensioned high strength bolts. Some recommendations are made as to the plastic design of the component parts of such an assembly and general conclusions are drawn which may affect the plastic design of many types of bolted connexions.

Notation

W_p = plastic collapse load on beams.
W = applied load on beams.
δ = vertical deflexion of free end of beam.
b = width of beam flange.
t = thickness of beam flange.
f_y = yield stress of beam material.
A = net cross-sectional area of bolt.
σ_y = proof stress of bolt material.
M = bending moment.
M_p = fully plastic bending moment.
L = distance between bolts.
B = width of end plate.
T = thickness of end plate.
f_w = " effective " yield stress of plate material.
τ = shear stress.
I_f = second moment of area of column flange.
t_w = thickness of web.
μ = Poisson's ratio.
f_y' = yield stress of column web.
P_y = yield load of the column web.
A_r = area of reinforcement to column web.
M_{pr} = reduced full plastic moment.
M_f = full plastic moment of beam flanges.
M_w = full plastic moment of beam web.
t_f = flange thickness.
d = distance between flanges.
V = applied shear force.
l = length of beam.
EI = bending stiffness of beam.

Introduction

The behaviour of bolted beam to column connexions has been investigated by Schutz (1959) and Johnson, Cannon and Spooner (1959). Whereas Schutz has discussed methods of analysis, the investigation of Johnson *et al.* has aimed at showing the adequacy of a variety of connexions employing high strength bolts in tension and shear. Recently, Charlton (1960) has shown that bolted joints, using high strength bolts in tension, will develop full strength when incorporated in a portal frame.

In the plastic method of design it is often necessary to position connexions at points of high moment. In consequence, it is advisable to devise methods for proportioning bolted connexions to develop the full plastic strength of the connected members. It has been the practice in the past to design the connexion to remain elastic at maximum moment, thus forcing plastic hinges to develop in the connected members. This may be safe design practice, but it is inefficient since little can be learnt as to the true behaviour of the connexion and much wastage of material results. A more rational approach would be to design the components of the connexion such that the entire assembly failed at maximum moment together with the connected members.

A second consideration in the plastic method of design is the requirement of rotation capacity. Since the failure of an entire structure is associated with a mechanism of discrete plastic hinges, the connexions must be sufficiently flexible such that the connected members may rotate at full plastic moment while hinges form elsewhere within the structure. In brief, the connexion must satisfy the dual structural conditions of strength and rotation capacity.

The current investigation is directed toward examining the behaviour of a single type of bolted connexion. The assembly consists of a plate profile welded to the ends of the beams and bolted to the flanges of the column with pretensioned high strength bolts, stiffening being provided to the column web in the regions of the beam flanges. This type of assembly has been selected from the tests conducted by Johnson as being the one which offers the greatest economy and simplicity. Schutz has also pointed out that plastically designed joints which make use of high strength bolts in tension require a smaller number of bolts and less fitting material than conventional splices which employ the bolts in shear only.

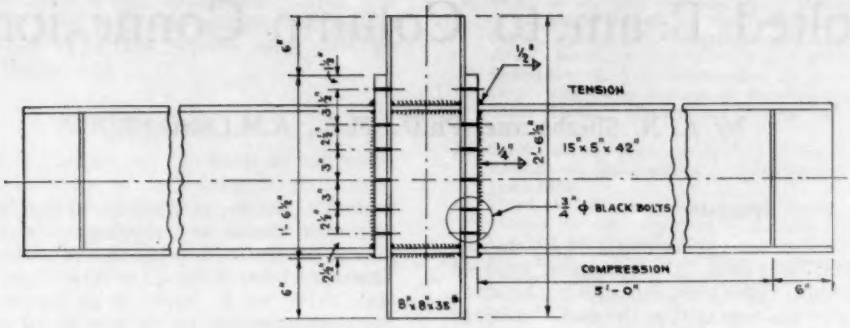
Attention has been paid to the strength of the three component parts of the connexion, viz.,

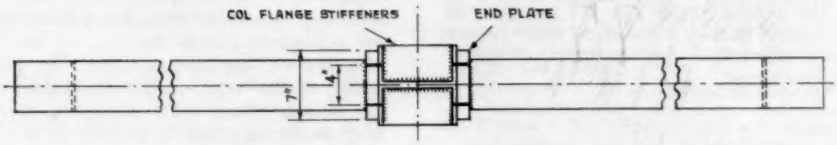
(A) The high tensile bolts.
(B) The end plate.
(C) Column stiffening in the region of the beam compression flange.

The Experimental Programme

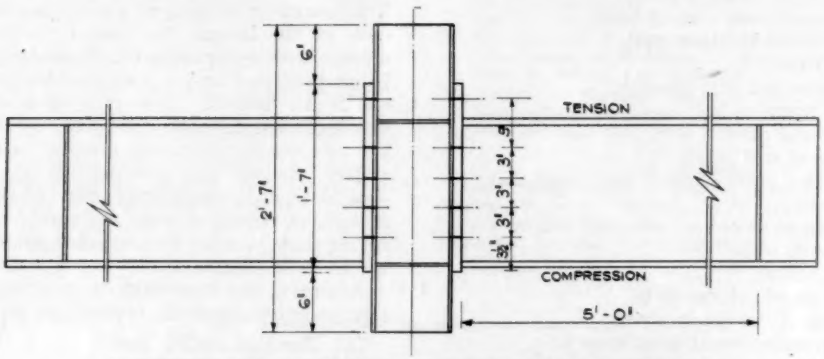
All the test specimens were fabricated from ordinary mild steel to B.S. 15-1948. The beams were 15 × 5 R.S.J. and the column stubs 8 × 8 × 35 lb. Universal Column Section. A series of five tests were carried out, the first group of three tests using $\frac{3}{4}$ in. diameter high tensile bolts in conjunction with $\frac{3}{4}$ in. diameter black bolts as shown in Fig. 1. The second series of tests, also shown in Fig. 1, used $\frac{7}{8}$ in. diameter high tensile bolts, when $\frac{3}{4}$ in. bolts were found to be inadequate for the size of beam involved.

Various combinations of end plate and column stiffener were employed in the tests. The end plate was kept at a constant width of 7 in. and the thickness ranged from $1\frac{1}{4}$ in. to $\frac{3}{4}$ in. Stiffening was provided to the column web in the region of the beam tension and compression flanges. The stiffeners spanned the full width of the column flange and varied in thickness from zero to $\frac{5}{8}$ in., the nominal thickness of the beam flange.





TEST	STIFFENER	END PLATE.	No OF ¾" φ H.T. BOLTS.	No OF ¾" φ BLACK BOLTS.	TYPE OF FAILURE.
A1.	t = 0	1¼"	16	8	COL WEB YIELDING & BUCKLING.
A2.	t = 5/16"	1¼"	16	8	BOLT FAILURE.
A3.	t = 5/8"	¾"	16	8	BOLT FAILURE.



TEST	STIFFENER	END PLATE	⅞" φ H.T. BOLTS	TYPE OF FAILURE.
B1.	t = 5/16"	1"	20 No.	LATERAL INSTABILITY OF BEAMS & LOCAL BUCKLING OF COL STIFFENER.
B2.	t = ½"	¾"	20 No.	WELD FAILURE AT BEAM TENSION FLANGE & END PLATE.

Fig. 1—Details of connexions

The bolts used in the test were ¾ in. and ⅞ in. diameter × 3 in. long high tensile bolts supplied to A.S.T.M. Standard A.325-55 T with a specified minimum proof stress of 37·9 tons per sq. in. and a minimum tensile strength of 53·5 tons per sq. in. They were used in the " as received " condition with hardened steel washers under both nut and head. Typical stress-strain curves for these bolts are shown in Fig. 2. These data were selected from tests on several bolts, there being some variation in the mechanical properties of the specimens tested. The stresses and strains were obtained from readings of the load and extension using a 2 in. gauge length and the net area of the bolt at the root of the thread. The gauge length of 2 in. was selected as being nearly equal to the grip length in all the tests. The handbook values of the net areas of the bolts were used to obtain the nominal stress.

A simple torque spanner was used to tighten the bolts, the torque being read off a gauge integral with the head of the spanner. The spanner was calibrated initially and found to be correct at torques of 320 lb. ft. and 470 lb. ft., the working torques of the ¾ in. and ⅞ in. bolts respectively. The calibration curve is shown in Fig. 3 over the entire range of the spanner. All traces of rust and mill scale were removed from the contact surfaces by wire brushing and the bolts were given an initial torque of approximately one-

third of the working value in order to bed the plates down. The nuts were then slackened off in turn, screwed finger-tight and finally brought up to the full working value of torque. In all tests the torque was applied to the nut, the bolt head being gripped to prevent slipping. The sequence of tightening began with the outermost tension bolt and continued down the row of bolts along each flange until the bolt nearest the compression flange was reached.

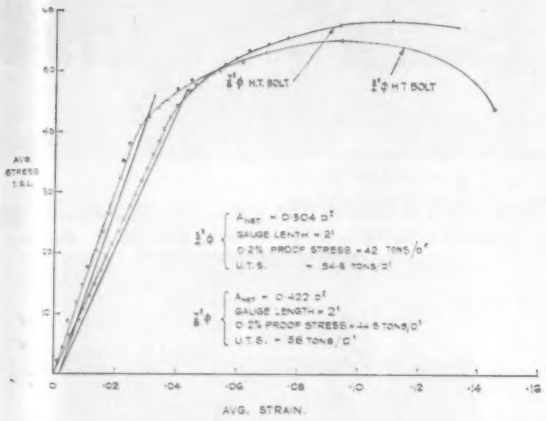


Fig. 2—Stress-strain curves for $\frac{3}{4}$ in. and $\frac{7}{8}$ in., ⌀H. T. bolts

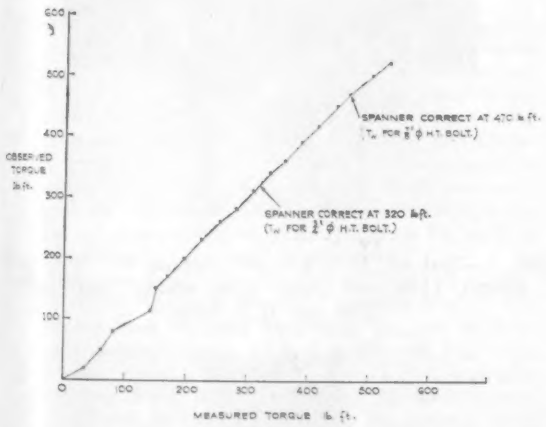

OBSERVED TORQUE lb. ft.
MEASURED TORQUE lb. ft.
SPANNER CORRECT AT 470 lb ft.
(T_m FOR $\frac{7}{8}''\phi$ H.T. BOLT.)
SPANNER CORRECT AT 320 lb ft.
(T_m FOR $1\frac{1}{4}''\phi$ H.T. BOLT.)

Fig. 3—Calibration of torque spanner

The arrangement of the loading and the instrumentation is shown in Fig. 4, and the tests were carried out in a 500 ton Amsler Hydraulic Testing Machine. The beams were loaded at a distance of 5 ft. from the face of the end plate and the column stub was allowed to rotate freely in space. The distance of 5 ft was chosen to avoid premature lateral instability of the beams, yet be sufficiently far from the connexion to minimise the effect of shear forces and obtain a condition of almost pure moment at the face of the column. In computing the plastic collapse load of the beam section near the end plate due allowance was made

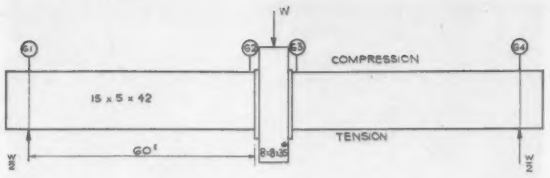


Fig. 4—Testing arrangement

for the shear forces. The modification of the full plastic moment due to shear was found to be negligible.

The area around each connexion was coated with ordinary plumber's resin to observe the initiation and propagation of yielding in the specimen.

Before beginning the series of tests on the connexions, a number of tension specimens were cut from the beam and column sections in order to determine the mechanical properties of the material. The results are contained in Table 1 and average values of the measured yield stresses were used in predicting the ultimate load of the connexion.

Discussion of Test Results

The test results for the five connexions are summarized in Fig. 5 in terms of the total applied load W and the average vertical deflexion δ of the ends of the beams. In all cases the experimental data are compared with the idealized load-deflexion characteristics for a 15 × 5 × 42 lb. R.S.J. as derived in Appendix I. Also included in Figs. 6 to 10 are photographs showing the connexions after test and Fig. 11 shows a typical weld failure which might occur in connexions of this kind. The photographs give a qualitative picture of the yielding in the component parts of the connexion at failure.

Throughout the tests the emphasis was on the overall behaviour of the connexion as typified by strength and rotation capacity. No attempt was made to determine the stress or strain distributions in the stiffeners, bolts or end plates.

Before discussing each test in detail, a few general comments may be made on the results of the test programme as a whole.

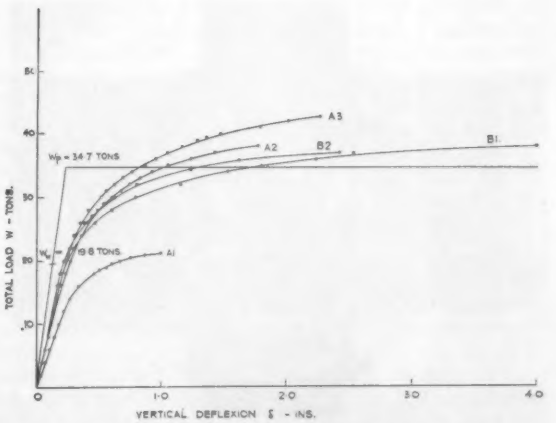


Fig. 5—Total load vs end deflexion—series A and B tests

Table 1. — Tension Test Specimens

Mark	Avg. Width in.	Avg. Thickness in.	Avg. Yield Stress Ton/sq. in.	Avg. U.T.S. Ton/sq. in.	Modulus of Elasticity E Ton/sq. in.
A1CF	1·001	0·488	15·8	28·45	12,800
A1CW	1·002	0·304	16·9	28·8	13,200
A1BF	1·002	0·668	15·55	29·6	12,800
A1BW	1·002	0·412	15·8	30·5	12,500
A2CF	0·999	0·484	—	29·2	12,800
A2CW	1·001	0·300	17·8	28·3	13,000
A2BF	1·001	0·661	16·0	31·0	13,800
A2BW	1·001	0·422	—	30·9	13,800
A3CF	0·999	0·464	16·65	29·4	12,700
A3CW	0·999	0·297	18·2	28·8	12,500
A3BF	0·998	0·665	16·5	29·4	13,200
A3BW	0·999	0·426	16·4	30·0	13,800

Avg. yield stress for the beams $f_y = 16{\cdot}0$ ton/sq. in.
Avg. yield stress for the column web $f_y = 17{\cdot}63$ ton/sq. in.

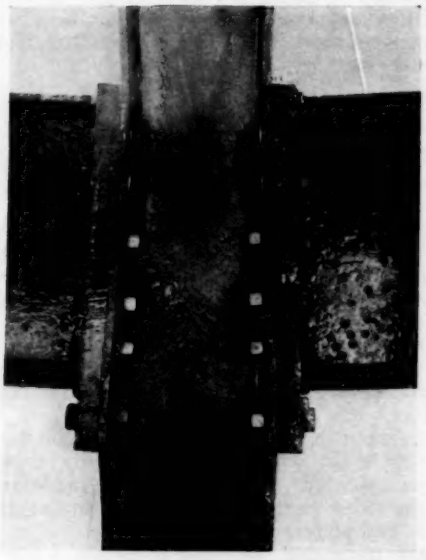


Fig. 6—Test A1. 15 × 5 × 42# beams. 8 × 8 × 35# column. 1¼ in. end plate. No column stiffeners. ¾ in. dia. H.T. bolts.

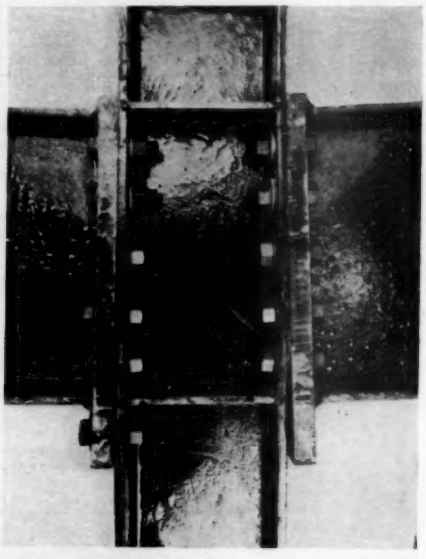


Fig. 7—Test A2. 15 × 5 × 42# beams. 8 × 8 × 35# column. 1¼ in. end plate. 5/16 in. column stiffeners. ¾ in dia. H.T. bolts.

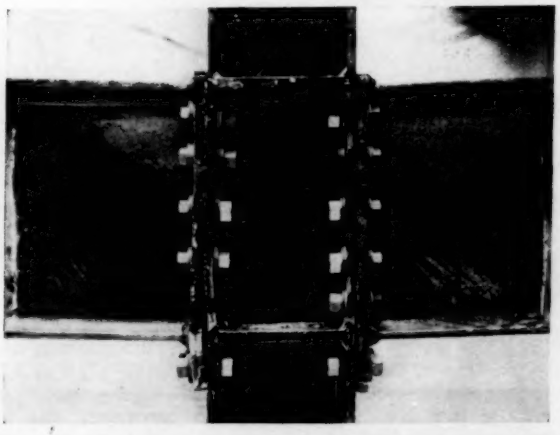


Fig. 8—Test A3. 15 × 5 × 42# beams. 8 × 8 × 35# column. ¾ in. end plate. 5/16 in. column stiffeners. ¾ in. dia. H.T. Bolts.

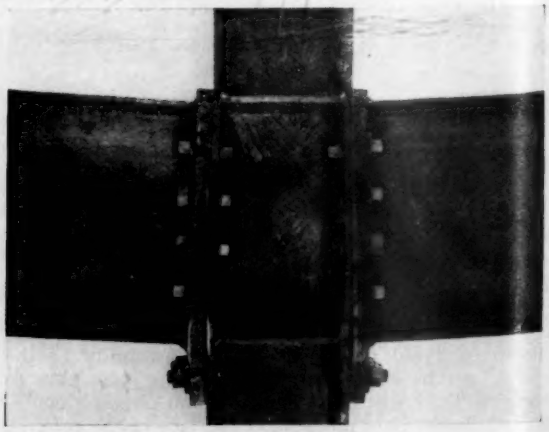


Fig. 9—Test B1. 15 × 5 × 42# beams. 8 × 8 × 35# column. 1 in. end plate. 5/16 in. column stiffeners. 7/8 in. dia. H.T. bolts.

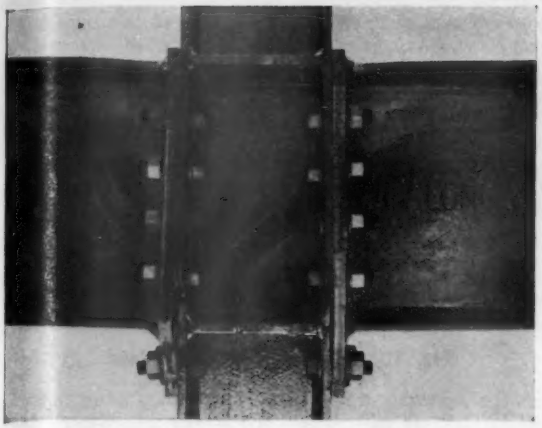


Fig. 10—Test B2. 15 × 5 × 42# beams. 8 × 8 × 35# column. $\frac{3}{4}$ in. end plate. $\frac{1}{2}$ in. column stiffeners. $\frac{7}{8}$ in. dia. H.T. bolts.

Fig. 11—Test B2. Weld failure in tension flange of beam.

(a) The curves showed the flexibility of this type of connexion. Comparing the slopes of the elastic portions of the experimental and predicted curves indicates that the maximum restraint developed was approximately 70 per cent of full rigidity. This restraint appeared to be quite independent of the thickness of the column stiffeners, being substantially unchanged for a $\frac{5}{16}$ in. stiffener (specimen A2) as for a $\frac{5}{8}$ in. stiffener (specimen A3).

(b) All the connexions except A1 met the requirement of strength inasmuch as the plastic collapse load was attained and even exceeded. However, it was only connexion B1 which met the requirement of rotation capacity in that the plastic collapse load was sustained to large deflexions. Specimens A2 and A3 might have qualified as sound connexions if larger bolts had been used and bolt failure avoided. Specimen B2 showed better characteristics than specimen B1 until a premature weld failure occurred.

(c) The connexions—except A1—behaved elastically to within the vicinity of the working load on the beams and inelastic behaviour became evident only beyond this load. The working load was derived using a load factor of 1·75 on the plastic collapse load W_p. Specimens A2 and A3, in which either the end plate or the column web reinforcement was very stiff, appear to have strain hardened, whereas specimens B1 and B2, which used a nominal amount of column reinforcement in conjunction with a moderately stiff end plate, exhibited considerable ductility insofar as the curve showed a flatter load-deflexion characteristic.

(d) Where the end plates were very stiff, i.e. connexions A1 and A2, the distortion of the column flange suggests an elastic stress distribution in the bolts. Where thinner plates were used such that bending of the plate was permitted, it appeared that the greatest distortions of the column flange occurred at the bolt immediately above the beam tension flange. This suggests an equalization of the forces in the bolts on either side of the beam tension flange.

(e) The photographs show relatively little yielding on the tension side of the beams although considerable yielding has occurred in the flange and web on the compression side. This would indicate that the connexion is fairly stiff to resist compressive forces but fairly flexible in the column flanges to absorb the tensile forces transmitted by the beam. Clearly the full plastic moment is resisted by the compressive forces in the beam at the face of the end plate acting in concert with the tensile forces in the bolts below the neutral axis.

(f) In the A series of tests the neutral axis appears to be located between the second and third bolts from the top, i.e. the beam compression flange. In the B series of connexions—which employ a different pattern of bolts—the neutral axis appears to coincide with the second bolt from the beam compression flange.

Test A1

No stiffening was provided to the column and consequently the failure was by local yielding and buckling of the column web between the beam compression flanges at a total load of 21·2 tons (0·61 W_p). A very heavy end plate was used in this connexion and it appears that the beam and end plate rotated together as an integral whole, all distortions being taken up by the column flanges. The maximum thrust in the column web appeared to be centred along the axis of the column at a point slightly below the line of beam compression flanges. No yielding was noticeable in either the beams or end plates.

Test A2

The effect of introducing a nominal amount of column stiffening can be observed by comparing this test with the previous specimen A1. Providing a stiffener only one half as thick as the beam flange was responsible for increasing the load capacity of this connexion to 38 tons (1·1 W_p) but the rotation of the connexion was curtailed by a failure in tension of one of the outermost bolts. This type of failure may be ascribed to the stiffness of the end plate in relation to the rest of the assembly. From Fig. 7 it can be

seen that the plate was sufficiently thick to resist bending between the bolts on either side of the beam tension flange, and this may have had the effect of inducing an elastic stress distribution in the bolt group. In consequence the higher stresses were carried by the outer line of bolts and no equalization of stress was possible.

Test A3

This test was designed to observe the effect of heavy column stiffening and a light end plate, using the same pattern of bolts as in the two previous tests. The connexion developed a maximum load of 42·8 tons (1·23 W_p) until failure occurred in one of the bolts immediately above the beam tension flange (Fig. 8). It may be concluded from such a failure that some equalization of the stresses took place in the bolts on either side of the beam tension flange. It would appear from the photograph that considerable yielding occurred in the end plate at the junction with the beam tension flange.

Test B1

This connexion developed the most favourable moment-rotation characteristics of the entire series inasmuch as the collapse load was sustained to large vertical deflexions of the ends of the beams. The maximum load carried was 38·2 tons (1·1 W_p) at a deflexion of 4 in. and failure only came about owing to the lateral instability of the beams after considerable yielding had set in along the beam compression flanges near the end plate. Unlike the previous tests, a moderately stiff end plate was used in conjunction with light column stiffening and this test best demonstrates the plastic behaviour of this type of connexion. An inspection of Fig. 9 shows that, at failure, considerable yielding has occurred in all the component parts of the assembly, viz. the beams, column stiffener plates and end plate, and considerable distortions of the bolts have taken place.

Test B2

The connexion was similar to connexion B1 except that a slightly thinner end plate and slightly thicker stiffener was used. The initial load-deflexion characteristics showed a marked improvement over specimen B1, the connexion being slightly stiffer and attaining the predicted collapse load at a smaller deflexion. However, premature failure of this connexion resulted from a tear in the fillet weld at the junction of the beam tension flange and end plate at a total load of 37 tons (1·07 W_p) and an end deflexion of just under $2\frac{1}{2}$ in. Again, as in previous tests, considerable yielding occurred in the compression fibres of the beams and the greatest distortions of the column flange were centred around the bolts immediately above the beam tension flange.

Design Recommendations

A few simple rules can be formulated for the plastic design of the component parts of the assembly. In general, these rules have been found to give conservative estimates of the load carrying capacity of the bolts, the end plates and the column stiffening.

(*a*) *The High Tensile Bolts*

At failure, it is assumed that the bolts above and below the beam tension flange are stressed equally. Implicit in this assumption is the fact that the end plate is allowed to deform and so bring about this equalization of load in the bolts. The four bolts

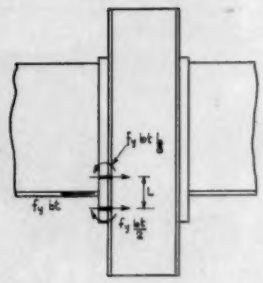


Fig. 12—Forces on the end plate.

can thus be designed to develop the strength of the beam tension flange, i.e.

$$btf_y = 4A\sigma_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where btf_y = the load carrying capacity of the beam tension flange.

A = net cross-sectional area of a bolt,

σ_y = proof stress of the bolt material.

Using handbook values for a 15 × 5 × 42# R.S.J., the required area of a bolt is given by :

$$A = \frac{bt}{4} \cdot \frac{f_y}{\sigma_y} = \frac{5 \times 0{\cdot}647}{4} \cdot \frac{16}{38} = 0{\cdot}341 \text{ in.}^2$$

which shows that a $\frac{3}{4}$ inch diameter bolt ($A_{net} = 0{\cdot}304$ in.^{2}) is inadequate and a $\frac{7}{8}$ inch diameter bolt ($A_{net} = 0{\cdot}422$ in.^{2}) is required.

(*b*) *The End Plate*

The size of end plate can be determined by considering the bending of the plate between the bolts on either side of the beam tension flange. Assuming the bolts are spaced an equal distance about the beam tension flange and that they completely restrain the plate, then the maximum bending moment developed in the plate is given by (see Fig. 12) :

$$M_{ext.} = f_y\, bt \frac{L}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where L is the distance between the bolts.

The internal resisting moment of the plate can be given by

$$M_{int.} = \frac{BT^2}{4} \cdot f_w$$

where B = width of the end plate

T = thickness of the end plate

$\frac{BT^2}{4}$ = plastic modulus

f_w = reduced yield stress due to the presence of shear stresses in the plate.

Applying a Mises Yield criterion the resisting moment becomes

$$M_{int.} = \frac{BT^2}{4}\sqrt{f_y^2 - 3\tau^2} \quad . \quad . \quad . \quad . \quad (3)$$

where τ is the average shear stress in the end plate and is given by

$$\tau = \frac{f_y bt}{2} \cdot \frac{1}{BT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equating the internal and external moments and substituting for τ obtain

$$f_y bt \frac{L}{8} = \frac{BT^2}{4}\sqrt{f_y^2 - \frac{3f_y^2}{4}\left(\frac{bt}{BT}\right)^2} \quad . \quad (5)$$

Equation (5) assumes that the yield stresses of the plate and beam material are identical. The equation can be simplified to yield

$$\left(\frac{bt}{BT}\right)^2\left[\left(\frac{L}{2T}\right)^2+\tfrac{3}{4}\right]=1 \quad \cdot \quad \cdot \quad \cdot \quad (6)$$

from which the plate thickness can be obtained.

Using handbook values for the dimensions of the beam and a value of 7 in. for the width of the end plate, the end plate thickness of the two series of tests can be deduced as follows :

A series $L = 3\frac{1}{2}$ in. $\quad T = 0{\cdot}89$ in.

B series $L = 5$ in. $\quad T = 1{\cdot}1$ in.

(c) *Column Stiffeners*

For most column sections used in practice, stiffening becomes essential in the vicinity of the compression zones of the column web if general yielding and instability is to be avoided. Parkes (1952) showed that the maximum stress in the web of an I beam under a point load was given by

$$f=\frac{2P}{3\sqrt{3}}Jt_w \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

where $J^3=\dfrac{(1+\mu)\,(3-\mu)}{2}\dfrac{I_f}{t_w}$

and I_f = second moment of area of the column flange

t_w = thickness of the column web.

The solution was derived by treating the column flange as a finite strip acted upon by a line load P and resting on an elastic foundation formed of the column web. The stresses in the column web were obtained from a stress function solution to the plane stress problem and the highest stresses were shown to occur at the web-flange junction immediately below the point of application of the load.

The yield load of the unstiffened column web may be determined by treating the end plate and column flange as a single strip of effective width $B = 7$ in. — the width of the end plate—and total thickness equal to the sum of the thicknesses of the two plate elements

$$P_y=\frac{3\sqrt{3}}{2}f_y't_w\sqrt[3]{\frac{(1+\mu)(3-\mu)}{2}\,\frac{B(t_f+T)^3}{12t_w}} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (8)$$

At full plasticity $\mu = 0{\cdot}5$ and the measured values of the column flange and web thickness are given by

$t_w = 0{\cdot}328$ in.

$t_f = 0{\cdot}492$ in.

The yield stress for the column web is taken from Table 1, $f_y' = 17{\cdot}63$ tons/in.^{2}

Simplifying equation (8) obtain for the load capacity of the unstiffened column

$$P_y = 1{\cdot}4\,f_y'\,(t_f+T)\sqrt[3]{t_w^2B}=22{\cdot}5\,(0{\cdot}492+T) \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (9)$$

If the beam flanges are assumed to resist the applied bending moment and the beam web to resist the applied shear, then the area of reinforcement required can be derived by providing enough material to develop the strength of the beam flanges. Assuming the yield stress of the reinforcement to be equal to that of the beams

$$btf_y = A_r f_y + 1{\cdot}4\,f_y'\,(t_f+T)\sqrt[3]{t_w^2B} \quad \cdot \quad (10)$$

For the five tests carried out a comparison is made in Table 2 between the theoretical and experimental load capacities of the connexion as measured in terms of the load applied on each beam. In computing the collapse load of the stiffened column web it has been assumed that the yield stress of the reinforcing plates is 16 tons/in.^{2} i.e. equal to that for the beams. It should be noted that a slight increase in the yield strength of the plate material would substantially increase the load capacity of the connexion, especially in the case of specimens A2 and B1 which employ thin plates.

Conclusions

From the results of this programme of tests it is possible to note certain features which may assist in the design of bolted connexions generally.

(a) A nominal amount of stiffening is usually required across the column web especially in the region of the compression flanges of the beams. The practice of providing a stiffener of identical thickness as the beam flanges can be quite inefficient and wasteful of material. The effect of this stiffener is two-fold inasmuch as the connexion gains in strength and attains the predicted plastic collapse load, and instability of the column web plate is avoided. This last item becomes important if deep column sections are used. Since the criterion for preventing instability in the column web is one of stiffness, not strength, a thin stiffener mounted horizontally and in contact with the column web would provide adequate bracing against out of plane movement.

Table 2.—Comparison Between Theoretical and Experimental Results

Exptl. Collapse Load on Each bm.	H.T. Bolts		End Pl. B=7″		Col. Web Stiffeners—Load Capacity					Type of Failure.
	Theor. Bolt dia.	Exptl. Bolt dia.	Theor. End Pl.	Exptl. End Pl.	Col. Web Capacity Tons Eqn. (9)	Area of Reinf. 7.672″ ×	Reinf. Capacity = (A × 16 tsi) Ton	Total Capacity P Ton	Theor. Load on each Beam. $=\frac{P\times\text{Depth of Beam }15''}{\text{Lever Arm }60''}$ Ton	
W/2 Ton	in.	in.	T in.	T in.						
10·6	$\frac{7}{8}$	$\frac{3}{4}$	0·89	1·232	38·8	—	—	38·8	9·7	Collapse of Col. Web.
19·0	$\frac{7}{8}$	$\frac{3}{4}$	0·89	1·232	38·8	0·318	39·0	77·8	19·45	Bolt Failure in Tension.
21·4	$\frac{7}{8}$	$\frac{3}{4}$	0·89	0·735	27·6	0·632	77·6	105·2	26·3	Bolt Failure in Tension.
19·1	$\frac{7}{8}$	$\frac{7}{8}$	1·1	0·975	33·0	0·318	39·0	72·0	18·0	Local Buckling of Col. Stiffener & lateral instability of beams.
18·5	$\frac{7}{8}$	$\frac{7}{8}$	1·1	0·735	27·6	0·529	65·0	92·6	23·15	Weld Fracture.

(b) The percentage restraint developed by this particular type of connexion appears to be quite independent of the thickness of the stiffener plates. In four of the five tests carried out the stiffening varied between one-half and full thickness of the beam flanges with little variation in the rigidity of the connexion.

(c) By far the most important conclusion concerns the plastic design of the entire assembly. For the connexion to be most efficient the component parts must be proportioned so that they reach the required strength and all yield simultaneously. Only in this way can full advantage be taken of the ductile character of mild steel, and only in this way will the requirement of rotation capacity be realised.

(d) The detrimental effect of providing a very thick end plate cannot be too strongly emphasised since such a plate may seriously restrict the rotation capacity of the connexion by its inability to deform plastically. In some instances a thick end plate may also reduce the strength of a connexion by imposing an elastic stress distribution in the bolts with consequent failure in tension of bolts along the outermost fibres of the end plate. It should be remembered that high tensile bolts have brittle mechanical properties and do not possess the ability to deform plastically and thus equalise the forces within a bolt group. This latter effect can only be achieved by proportioning the thickness to permit plastic deformation of the plate.

Acknowledgements

The work described in this report was carried out in the Engineering Laboratory of the University of Cambridge, under the general direction of Professor J. F. Baker. It forms part of a general investigation into the behaviour of bolted and welded connexions being carried out with the assistance of the British Welding Research Association.

The author would like to thank members of the workshop staff for fabricating the test specimens, and Messrs. R. W. Clark, J. W. Gatiss, J. D. H. Morgan, R. A. D. Noble and C. P. Woodcock, undergraduates in the Department of Engineering, for assistance in designing the experiments and carrying out the tests.

Appendix

The theoretical load-deflexion relationship for a $15 \times 5 \times 42$ lb. R.S.J. can be determined on the basis of an idealized elastic-perfectly plastic stress-strain curve for mild steel. The predicted collapse load for the connexion can be obtained from the value of the fully plastic moment of the joist, suitably modified for the effect of shear forces

$$M_p{}^r = M_f + \sqrt{M_w\left(M_w - \frac{3V^2}{4t_w f_y}\right)} \quad \text{(a) Baker et. al. (1956)}$$

$M_p{}^r$ = reduced plastic moment
M_f = full plastic moment of the flanges alone in the absence of shear forces $= bt_f(d + t_f)f_y$
M_w = full plastic moment of the web alone in the absence of shear forces $= \frac{t_w d^2}{4} f_y$
t_w = web thickness
t_f = flange thickness
d = depth of section measured between inside edges of the flanges
f_y = yield stress
V = applied shear force $\frac{W}{2}$ (see Fig. 4)

The above relationship is derived from a Mises Yield criterion and assumes that the flanges carry the bending, that the web carries the shear and is limited to

$$V = \frac{W}{2} \leqslant \frac{t_w d f_y}{\sqrt{3}} .$$

Using handbook values for the section properties of a 15×5 R.S.J. and a yield stress $f_y = 16$ tons/in.^{2} from Table 1 obtain

M_f = 743 ton in.
M_w = 316 ton in.
M_p = 1059 ton in.

Equating the reduced plastic moment to the applied moment obtain

$$\left(60\,\frac{W}{2} - 743\right)^2 = 316\left(316 - \frac{W^2}{35{\cdot}84}\right) \quad \text{(b)}$$

from which the total collapse load $W = 34{\cdot}7$ tons.

Check $\frac{t_w d f_y}{\sqrt{3}} = 52{\cdot}9$ tons. i.e. $\frac{W}{2} < \frac{t_w d f_y}{\sqrt{3}}$.

Compare with the load required to produce the full plastic moment $M_p = 1059$ ton in.

$$\frac{W}{2} \cdot 60 = 1059$$

$$W = 35{\cdot}3 \text{ tons.}$$

The modification due to shear force is very small.

The end deflexion of each beam can be obtained by assuming the connexion to develop full restraint

$$\delta = \frac{W}{2} \cdot \frac{l^3}{3EI} .$$

For $E = 13{,}400$ tons/in.^{2}
$I = 428{\cdot}5$ in. (Handbook value.)
$\frac{W}{2} = 17{\cdot}35$ tons.

obtain $\delta = 0{\cdot}22$ in.

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Discussion

The Council would be glad to consider the publication of correspondence in connexion with the above paper. Communications on this subject intended for publication should be forwarded to reach the Institution by September 30th, 1961.

Structural Considerations of a New Animal Feed Mill in Belfast, Northern Ireland*

Discussion on the paper by Edward V. Finn, M.I.Struct.E., A.M.I.C.E., and Kenneth F. Shadbolt, A.M.I.Struct.E.

THE PRESIDENT welcomed the visitors to the meeting, and introduced the authors.

Mr. FINN then presented the paper.

Discussion

Mr. F. S. SNOW, C.B.E., (Past President), congratulating both authors on their presentation of the paper, said that, in his opinion this descriptive type of paper was very valuable and most interesting.

The authors would be the first to say they could not carry out such jobs as had been described without having the help of a good team and they would also agree that they had a wonderful team working on this job.

In congratulating the Contractors on their work in the construction of the new mill at Belfast, Mr. Snow welcomed Mr. Campbell, the Chairman of Messrs. McLaughlin and Harvey and said that one could not wish to work with better contractors anywhere. They had had vast experience and had given the greatest co-operation. He believed the contractors' account had been settled within three months after the mill opened and all Consultants and Clients knew that this was of great advantage to all parties.

Expressing his view on the recognition of engineers and architects, Mr. Snow said that on opening days it had always appeared to him that everybody except the engineers were congratulated on " the wonderful job they had produced."

Northern Ireland had its unemployment problems. The services of engineers and contractors from this country were required all over the world ; as a result of the works described in this paper and the work described by Mr. Scruby (Member of Council) in an earlier paper entitled " Structural Problems connected with Palletization " some 500 people of Northern Ireland had subsequently found permanent employment.

It was as well to appreciate the importance of the engineering profession not only to Northern Ireland and the rest of Great Britain but to countries all over the world. There was no doubt that the siting of the new animal feed mill at Belfast had made an appreciable contribution to the economy of Northern Ireland.

Mr. S. THOMPSON (Managing Director, Associated Feed Manufacturers, Ltd.) paid his tribute to the engineers and contractors responsible for the design and construction of the new mill ; everyone concerned had worked very well, and it would be invidious to refer to any one of them. But he added his Company's thanks to Mr. Possnett for his fine work in dealing with the initial arrangements.

When looking at the published plans of the site, he continued, one might be struck by the large open spaces left around the buildings. They had been provided quite deliberately, because all the products coming into and all the finished products going out from the mill were transported by road, and flexibility and mobility were required for the movement of traffic in and out. There were something like 400 vehicles per day in and out.

Mr. H. E. CAMPBELL (Managing Director, Messrs. McLaughlin and Harvey) thanked the Institution for the opportunity to speak in connexion with the paper.

As the representative of the main contractors, he congratulated the authors on the condensation of so much information on such a large volume of work in so few words. That might give rise to difficulty in reading the paper, and there was the danger that some point might not receive the elaboration which it would otherwise deserve.

In that connexion, the note regarding piling to the silo (at the bottom of Column 1 on page 301) required some amplification. In this case Messrs. Christiani and Nielsen were the specialist designers and contractors for the silo and were responsible for the foundations. His firm became sub-contractors for the piling, to be carried out to the requirements of Messrs. Christiani and Nielsen. The piles, which in many cases were arranged in groups at 3 ft. 6 in. centres, were driven in accordance with the usual practice for Vibro piles.

The driving sequence was arranged to give the maximum time lag between the driving of adjacent piles. When the 294 piles were driven, four piles were tested, and from their behaviour it was soon realised that in the particular ground conditions at the site the driving of the last pile in a group at close centres caused uplift of the piles already driven in the group. This trouble had been known to occur with both precast and timber piles, but it had not been met by his Company in 30 years experience with in-situ piles. Confirmation of what had happened was given by the fact that no settlement was found in the test of a final pile of a group.

Investigation of the matter having been made, it was then decided to pre-load to 100 tons thus making ample provision for a working load of 60 tons. Kentledge to a total weight of 120 tons was built on four 30-ton bogies connected by a steel frame and set on two sets of rails.

By this means it was possible to move the kentledge along the line of the piles and, following preparation at the head of each pile, the load to a minimum value of 100 tons was applied and checked for no settlement. By this means it was possible to carry out the operation with the minimum of delay. The actual testing operation on the 294 piles had taken a little under six weeks.

The warehouse foundations provided an opportunity to examine the condition of the sandstone in-situ as excavation was carried down to the planned level,

* *Read before the Institution of Structural Engineers at* 11, *Upper Belgrave Street, London, S.W.*1. *on the 27th October*, 1960, *Lt.-Col. G. W. Kirkland, M.B.E.(Mil.), M.I.Struct.E., M.I.C.E., (President) in the chair. Published in " The Structural Engineer " Vol. XXXVIII, No.* 10. *pp.* 298-309.

It was found that the decomposed surface of the sandstone extended for a depth of about 4 ft. The effect of rain or flooding on the decomposed material quickly turned it into a sandy slurry. Exceptional care was taken to remove any decomposed rock and to seal the sound sandstone with a minimum delay after the surface had been finally trimmed and prepared.

A point which was not in the paper, but was mentioned by Mr. Finn, was the necessity to arrange for the installation of large items of permanent plant during and in conjunction with the construction. The photograph of the production building steel frame (Fig. 7) shows mixers, cookers and other important items of permanent plant placed in position when the first three bays of structural steelwork had been finally lined up. Co-operation and careful planning were not only desirable, but most essential in the carrying out of a combined operation of this sort, and the fact that it had contributed in no small way to the achievement of a record time of completion would surely indicate that the Consulting Engineers had to overcome many problems involving human relationships as well as those of a more highly technical and definite nature.

Mr. H. CUNNINGHAM (Resident Engineer, Frederick S. Snow and Partners) said it was evident from the authors' illustrations that there was from the start of the job a clear cut programme involving civil works, structural, building and finishes. It was also a matter of the greatest urgency to complete a stage of construction where the electrical and mechanical contractors would be able to carry out their installation. Owing to the considerable amount of constructional equipment on the site, close co-operation was required between contractors, to achieve the programmed target dates.

Some of the quantities used on the job were as follows :—

Bricks	1,021,000
Hardcore	34,500 cu. yds.
Piles	550
Concrete	18,200 cu. yds. (not including silo)
Structural Steel	1,023 tons.
Reinforcement	800 tons.
Patent glazing	60,000 sq. ft.
Ordinary glazing	12,000 sq. ft.

Mr. F. LISSAU (Copenhagen), who was invited to speak, said it had been a great experience, to work in Northern Ireland, there had been good co-operation between all those engaged in the work and he would take the opportunity to thank Mr. Campbell for the swift action he took when it was found that the piles did not stand up to the specification.

Mr. M. T. SHAW (Member) mentioned the reference in the paper to sleech overlying the sandstone, and asked if it were the blue sleech or blue alluvial clay.

With regard to the piling, he wondered whether there was any particular reason why there should be a change from prestcore to driven piles.

Finally, he asked whether many of the materials used on this project had to be imported into Northern Ireland.

Mr. F. R. BULLEN (Vice-President), commenting on the troubles experienced with piling, asked what was the nature of the material the authors had described as " sandstone."

He had been wondering whether the toes of the piles were driven down to the sandstone or were lying in the upper material, whatever it was. He would think that in such conditions the piles would have been driven down to the sandstone and would have become end bearing piles.

At what depth of piling or other foundation was it thought more economical to change from the one type to the other? It was sometimes thought that at depths below a certain number of feet, perhaps 12 ft., the pier became the more economic. That might explain why piles were used in one place and piers in other places.

Following Mr. Shaw's question on piling, Mr. Bullen asked why a variety of piles was used—whether the piles used were more readily available or were perhaps cheaper at the time the tenders were sent out.

Finally, were all the piles 60-ton piles? He asked for any data there might be in connexion with the testing of the piles, including deflexion diagrams and similar information.

Mr. K. H. BRITTAIN, V.R.D., (Associate Member) asked if there were any special requirements for perfect finish on the inner surface of the silo walls, and, if so, how those special requirements were met. He had in mind that in feeding stuffs bins there should be no air holes that could contain a certain amount of material which might decay.

Mr SHADBOLT replied to some of the questions.

First he apologised to Mr. Campbell for not having mentioned that Messrs. McLaughlin and Harvey, Belfast, were the sub-contractors for the piling.

The reference in the Paper, which Mr. Campbell had mentioned, to the trouble experienced with the piling had been made as a matter of interest, and the authors believed that this was the first time that a remark of this kind had occurred in print. Other people had since told them about piles in connexion with which the same trouble had arisen.

Presumably the question by Mr. Shaw concerning the importation of materials into Northern Ireland related especially to bricks. Well over a million bricks were used on the job. A really tough brick was required for the first 6 ft. 6 in. of construction height, and they had not come across any in Northern Ireland which had the same toughness as English engineering bricks. Most of the other materials were obtained locally and, indeed, every effort was made to use local materials, particularly having regard to the unemployment problem there, which Mr. Snow had mentioned.

The sleech overlying the sandstone is termed " blue." Its true classification is blue estuarine clay of very recent deposition, with high sand content.

In reply to Mr Bullen the geological classification of the sandstone is red bunter, triassic sandstone.

With regard to the piling he assumed that the questions referred to the change from Vibro piles under the silo to the prestcore piles under the boilerhouse. He felt sure that Mr. Lissau representing Messrs. Christiani and Nielsen, who designed and constructed the silo, could answer for this building better than he could, and that Mr. Finn would deal with the change in piling.

In reply to Mr. Brittain's question, he explained that the walls of the silo were precast and that a sufficiently smooth dense finish was obtained so that no specific treatment was therefore necessary.

Mr. FINN in a further comment on piling said that bored piles were used to support the edible oils building and the boilerhouse. These bored piles were driven much later in the contract and it was preferable to use

a light bore pile rig in this particularly soft area of the site. The bored piles were also capable of supporting 60 tons each.

Referring to Mr. Bullen's question concerning the depth of piers, he said that the type of foundations chosen was based on an economic consideration and for this site, which had a great many existing foundations and ducts well below surface level, a depth of approximately 17 ft. was found to be the critical depth. The depth would vary from site to site and he agreed that generally speaking, the figure of 12 ft. given by Mr. Bullen was normal.

Mr. Finn explained that indigenous materials had been used wherever possible but apart from engineering bricks previously mentioned, the asbestos cladding had also been imported, although this and other work had been erected by local contractors.

Mr. HERBERT (Frederick S. Snow and Partners) supported Mr Finn's remarks concerning the difference in the types of piles used on the job. The majority of the piles were covered by the Contract of Messrs. Christiani and Nielsen on the silo building and these were cast in-situ driven piles.

The smaller numbers of piles used for the boiler house building etc., were the prestcore piles which were more economical in these locations. A driven pile frame with the resultant cost of moving and waiting time on the frame would have meant more costly piles.

He confirmed that there were practical reasons for not using piling, as described in the paper, namely, that the depth and extent of obstructions on the site of the production and finished products buildings were such that piling would have been uneconomical.

An analysis was made, and showed that the depth was approximately 17 feet.

Mr. Finn commented that Mr. Herbert was the Quantity Surveyor on the project, with whom he had discussed this question.

Mr. A. F. POSSNETT (Technical Officer of U.K. Milling Group, Unilever Ltd.) joined in the discussion to correct an impression which might have been gained in connexion with the silos. He said the clients did specify the finish. Indeed, a major factor in the decision to use this particular type of silo was based on the better internal finish to be obtained with a pre-cast job than with an in-situ job using conventional sliding shuttering. The design of the silo, its fundamental conception for the satisfactory handling of the various raw materials, many of them having poor flow properties, was one of the most difficult problems facing the design team. For many years the Company had given a great deal of consideration to the basic design of silos for use in the animal food industry and the design adopted for this mill was the outcome, using the services of their very good friends Messrs. Christiani and Nielsen Limited to perfect the whole of the civil engineering features involved.

Mr. LISSAU said that the length of the piles ranged from 21 ft to 35 ft. Above the sandstone was sand, silt, and then a mixture of fill and old foundations.

The method of testing the piles was one commonly used by Christiani and Nielsen, that is, the pile is loaded and off loaded in stages up to the ultimate load, and this is considered as the load where the permanent settlement is 1·5 times the elastic settlement. The permissible load is taken as half the ultimate load.

The result of the first pile test gave rise to some discussion because it could have been accepted, using other criteria, but the three subsequent tests left nobody in doubt about what had happened with the piles.

During the "re-driving" of the piles they all followed a pattern of settlement which could be predicted from the original driving sequence and certain isolated standing piles, which were difficult to reach with the travelling kentledge, could be omitted.

THE PRESIDENT, at the conclusion of the discussion, expressed the thanks of the meeting to Mr. Shadbolt and Mr. Finn for the manner in which they had replied to the points raised.

Corrigendum

Under Fig. 16 in the paper, the word "waterhouse" should read "warehouse."

Book Review

The Analysis of Deformation, Vol. IV. "Waves and Vibrations," by Keith Swainger. (London : Chapman and Hall, 1959). 8 in. × 5½ in., 370 plus xxvii pp. 75s.

This is the fourth volume in a series of six—two of which have yet to be published—written with the aim of unifying the analysis of deformation of fluids and solids under the influence of static and dynamic loads. The volume is a study of the propagation of stress through, and on the surface of, substances possessing various physical properties such as elasticity, plasticity, fluidity or combinations of these. The text is written in terms of vector analysis and the multitude of symbols may appear at first sight rather disturbing, but the problem simply involves becoming familiar with the mathematical shorthand employed. The author comments critically on the theories and analyses of other investigators in the field of waves and vibrations and examines the available experimental evidence although, as he points out, there is a paucity of experimental data, particularly under controlled conditions.

The volume is highly specialized as far as structural engineering is concerned and will undoubtedly be of greatest service in research on dynamic stress propagation. There is an interesting chapter on geophysics of the Earth, arising from the analysis of earthquake waves and vibrations, but it should be observed that the views of geophysicists and of geologists are seldom completely reconciled on the subject of the structure of the Earth and crustal movement.

F.D.C.H.

Groups of Piles Under Mono-Planar Forces *

Written Discussion on the Paper by L. D. Turzynski, A.M.I.Struct.E.

Mr. I. L. GULYAS wrote that several logical mistakes appeared to have been made in the mathematical derivation of the equations used by the author. He also commented that using conventional eliminative methods of solution of equations the results arrived at by Mr. Turzynski could be arrived at with half the energy and time required by the methods of vector analysis used by the author. While welcoming the utilization of matrix notation for clarity in the solution of structural problems, it appeared that the author had shot at a sparrow with a cannon when a three millimetre air-gun would have been sufficient.

On page 287 the equation

$$\begin{bmatrix} v \\ h \\ m \end{bmatrix} = \begin{bmatrix} \cos\varnothing \\ \sin\varnothing \\ x\cos\varnothing \end{bmatrix} \begin{bmatrix} \cos\varnothing & 0 & 0 \\ 0 & \sin\varnothing & 0 \\ 0 & 0 & x\cos\varnothing \end{bmatrix}$$

appeared to be a mathematical impossibility, since

$\begin{bmatrix} \cos\varnothing \\ \sin\varnothing \\ x\cos\varnothing \end{bmatrix}$ is a one dimensional matrix while:

$\begin{bmatrix} \cos\varnothing & 0 & 0 \\ 0 & \sin\varnothing & 0 \\ 0 & 0 & x\cos\varnothing \end{bmatrix}$ is a three dimensional matrix.

To multiply the two was mathematically unimaginable.

On page 289 numerical values were substituted and the results were correct, but no one could detect how they were arrived at, since the multiplication indicated could not be carried out mathematically, and in matrix calculus one was unable to divide back the product to obtain the factors since vector division was not conceived.

Returning to equation (1) page 287, and supposing that the notation was correct the position of the two matrixes should be changed. In matrix calculus it was of primary importance whether a matrix was pre-or post-multiplied.

In the paper pre-multiplication was indicated, but from the result it appeared that post-multiplication had been carried out, i.e.

$$\begin{bmatrix} v \\ h \\ m \end{bmatrix} = \begin{bmatrix} \cos\varnothing & 0 & 0 \\ 0 & \sin\varnothing & 0 \\ 0 & 0 & x\cos\varnothing \end{bmatrix} \begin{bmatrix} \cos\varnothing \\ \sin\varnothing \\ x\cos\varnothing \end{bmatrix}$$

but even so it was logically clear that a row-by-column multiplication of a three dimensional matrix needed another 3-D matrix to make a product, and incidentally make sense.

Mr. TURZYNSKI, in reply, regretted that he could not agree with Mr. Gulyas. Neither ' several ' nor ' logical ' mistakes had been made. Inversion of matrices is quicker than solving simultaneous equations by elimination for the case where several load combinations have to be considered. He had not used vector analysis in deriving the final formulae. Mr. Gulyas must have jumped to several erroneous conclusions in making these statements.

In deriving the final formulae, the method of analysis used by Mr. Turzynski was the usual flexibility coefficient technique. In *presenting the results*, however (as stated in the paper) matrix *notation* was used for conciseness. It was because of this (i.e. working backwards) that an error of presentation occurred in the ' interim ' line above equation (1) in the paper. This line should read

$$\begin{bmatrix} v \\ h \\ m \end{bmatrix} = \begin{bmatrix} \cos\varnothing & 0 & 0 \\ 0 & \sin\varnothing & 0 \\ 0 & 0 & x\cos\varnothing \end{bmatrix} \begin{bmatrix} \cos\varnothing + \sin\varnothing + x\cos\varnothing \\ \cos\varnothing + \sin\varnothing + x\cos\varnothing \\ \cos\varnothing + \sin\varnothing + x\cos\varnothing \end{bmatrix}$$

This error of presentation affected neither the accuracy nor the form of equation (1) and subsequent work in the paper. As Mr. Gulyas stated, post-multiplication should be carried out in the above equations, but in his remaining comments on the matrix methods adopted—presumably the reference to ' logical ' mistakes—his views were erroneous for two reasons. Firstly, a three dimensional matrix *can* be post-multiplied by a column matrix (as is done above) and make sense. This is amply supported in text books on the subject. Secondly, the actual post-multiplication suggested by Mr. Gulyas (i.e. his last equation) is wrong, in that it results in the wrong answer.

Dr. E. C. SMITH (Member), congratulated the author on presenting a very interesting Paper, and suggested that the following points should be noted as much time could be saved by tackling the problem in the simplest manner.

Groups of piles under mono-planar forces could be classed as :

(1) Combinations which are statically determinate, and

(2) Combinations which are statically indeterminate.

All pile groups were statically determinate if there were not more than three rows of piles. (Fig. 8 of the Paper shows what is meant by a row of piles). When the number of rows exceeded three, the piles formed a statically indeterminate group.

Considering first statically determinate groups, the number of unknowns could be reduced to three, which were the axial loads on the piles. All that was necessary for equilibrium was that the following conditions be fulfilled :—

Condition I.—The Vertical components of the forces must balance.

Condition II.—The Horizontal components of the forces must balance.

Condition III.—The Moments of all forces about any point must balance.

The axial loads on the piles could then be readily found by solving the three simultaneous equations given by the above conditions.

*Published in " The Structural Engineer," Vol. XXXVIII. No. 9., p. 286. (September, 1960)

It was important to recognize whether the pile group was statically determinate or otherwise. Example 3 given by the author on Page 291 was for a statically determinate group, and the complete solution could be obtained by merely solving the three simultaneous equations given by the above conditions. In Example 3 there was no need to calculate the movement of the pile cap to find the axial forces on the piles.

If there were more than three rows of piles, the pile group was statically indeterminate. With statically indeterminate combinations, the three conditions given above had to be fulfilled and in addition the loads on the piles had to be consistent with the pile cap being rigid. It was interesting to compare the author's method with the method given by Krey in "Erddruck, Erdwiderstand and Tragfähigkeit des Baugrundes." Both methods gave the loads on each pile in terms of the movement of the pile cap and enabled three simultaneous equations to be obtained from which all the unknowns could be found. Krey, however, expressed the movement of the pile cap in terms of the horizontal displacement and the vertical displacements at the extreme ends of the pile cap, and that was a useful approach in some cases.

With statically indeterminate combinations it would be appreciated that any number of solutions could be found which gave axial loads on the piles for which the horizontal forces, vertical forces and moments respectively balanced, but there was only one solution which gave the axial loads such that, in addition to being in equilibrium, they were consistent with the pile cap being rigid.

A check to ascertain that the axial forces were such that their displacements were consistent with the pile cap being rigid gave useful information on the accuracy of the work.

Mr. TURZYNSKI appreciated Mr. E. C. Smith's comments and explanations regarding the basic differences between the statically determinate and indeterminate cases and the conditions which must be satisfied by the solutions for both.

Mr. Smith stated that a pile cap under mono-planar forces consisting of three rows is statically determinate. That was quite true, but if there were any raking piles in the group, then due to the rigidity of the pile cap the compatibility of strains, illustrated in section 2 of the paper, had also to be considered (i.e. in addition to the three conditions of equilibrium). This was not always remembered, but was taken care of automatically in the proposed method. For that reason, and the fact that the final result required the solution of only three simultaneous equations regardless of the degree of statical indeterminacy, no specific distinction had been made in the paper between the two types of pile groups. In practice, such a distinction might well be made.

Example No. 3 was used, as stated under 'General Remarks,' for the sole purpose of illustrating the method to be adopted when the longitudinal spacing of the individual piles in the group differs. As the proposed method was identical, whether the group was statically determinate or indeterminate, the simplest layout was used.

Mr. Turzynski realised the fact that there were many ways in which one could have defined the final position of the pile cap due to the applied loads, but chose the one which seemed the most obvious—reference to Krey's paper given by Mr. Smith was therefore helpful and welcome.

Corrigendum

In Fig. 6 (on page 289) the distances of piles No. 2 and No. 4 should read 3′ - 0″ and not 3′ - 6″ as shown.

Book Review

Stresses in Shells, by Dr. Ing. W. Flügge. (Berlin/Göttingen/Heidelberg: Springer-Verlag, 1960). 9¼ in. × 6 in., 499 plus xi pp. D.M. 58.80.

Most design engineers and research workers engaged in the field of shell structures will have made reference to Flügge's "Statik und Dynamik der Schalen" but this new work in English by the same author, having reached the shelves three years after an announcement of impending publication, should not be dismissed as a translation and revision of the earlier work. It must stand as one of the most comprehensive books on shell structures.

The book should satisfy to a great extent those practising engineers who are in need of guidance in applying shell theories to their problems, and also research workers who seek a record of well-established work in the field. The former will appreciate that the theoretical development of each case or structure considered has in general been followed out to a stage where the results are in a form suitable for direct application in design. In places the mathematical content of the work is perhaps beyond the average graduate engineer's experience but the author claims that in his treatment no advanced mathematical techniques have been unnecessarily introduced.

There are two main sections in the book. In the first shell are considered primarily as membrane structures while in the second the more complex but realistic conditions involving bending are analysed. The author shows how the simple though erroneous solution by the membrane theory can be made to indicate its own shortcomings.

The circular cylindrical shell receives a large share of the author's attentions but this is not at all at the expense of the general problem. In each main section detailed treatment is given also to shells of revolution and shells of arbitrary shape.

Under these general headings are included polygonal domes, folded structures, elliptic and hyperbolic paraboloids. A useful section discusses anistropic structures such as plywood shells. The buckling of cylinders is considered in some detail in a concluding section.

An excellent feature is the very full 13-page bibliography, classified according to the chapters in the book and made more useful by the inclusion of a brief comment on the theme of the references.

This is a big book (in both senses), clearly printed and easily read, and will doubtless become familiar to many engineers. It is certainly worth investigating.

A.R.D.

Institution Notices and Proceedings

ORDINARY GENERAL MEETING

An Ordinary General Meeting of the Institution of Structural Engineers was held on Thursday, 27th April, 1961, at 5.55 p.m. Lt.-Colonel G. W. Kirkland, M.B.E.(Mil.), M.I.Struct.E., M.I.C.E. (President) in the Chair.

The following members were elected in accordance with the Bye-Laws. Will members kindly note that the elections, as tabulated below, should be referred to when consulting the Year Book for evidence of membership.

STUDENTS

AH-KEE, Roger, of London.
ARNOTT, Edwin Howard, of Salisbury, Southern Rhodesia.
CRABB, Robert Thomas, of Southall, Middlesex.
FERGUSON, Colin, of Harrogate, Yorks.
FLETCHER, David Vincent, of Sheffield.
FOX, Terence George, of Pudsey, Yorks.
HEWETT, Peter, of Stanmore, Middlesex.
MANGTANI, Bhagwan Kundomal, of Bombay, India.
MANSFIELD, Christopher John, of Bilborough, Nottingham.
PATEL, Kanubhai Somabhai, B.E.(Civil), of Gujarat, India.
RABBITT, Denis, of Bolton, Lancs.
WILSON, Colin, of Bolton, Lancs.

GRADUATES

AFOLABI, Joseph Oduntan, of London.
ALDERSON, Christopher Dale, of Alresford, Hants.
ANDERSON, Alan Frederick, of London.
ARTHUR, Roland Eric Brough, of Bolton, Lancs.
BANDYOPADHYAY, Nirmalendu, B.E., of Calcutta, India.
BASU, Rabindra Nath, B.E.(Civil), of Calcutta, India.
BHAT, Kaje Rama, of Bombay, India.
BLACKBURN, Michael Peter, of St. Albans, Herts.
CHABLANI, Mohan Ghanshamdas, of Ahmedabad, India.
CHAN FOOK CHOY, of Singapore.
CHIU CHI SHING, B.Sc., of Wanchai, Hong Kong.
CHOW WAI CHEUK, of London.
CLARK, Brian Richard, of London.
COUSINS, Raymond John Randal, B.Sc., of London.
EMIABATA, Basiru Ademola, of London.
EYRE, David Alan, of Manchester.
FERNANDEZ, Osborne Anthony, of London.
GEE, Brian Lewis, B.Sc.Tech.(Hons.), of London.
GODFREY, Dan, of Purley, Surrey.
GRAHAM, Ian, of Reading, Berks.
GRIFFITHS, John Charles, of London.
HADEN, Thomas Roy, of Sale Moor, Cheshire.
HAQUE, Malik Israrul, B.Sc., of Karachi.
HO HOK LING, of London.
ISANEDIGHI, Okon Uwe, of London.
JOHNSON, Kenneth Richard, of Grays, Essex.
JOHNSON, Trevor William, of Ash Vale, Surrey.
JONES, Dennis Barker, of Liverpool, Lancs.
LAL, Achinta Kumar, B.Sc., of London.
LARNER, Raymond James, of Ilford, Essex.
MALES, Kenneth Sidney, of London.
MANIAR, Priyavadan Umedlal, B.E., of Ahmedabad, India.
OBIOZO, Leo Emmanuel, of Surbiton, Surrey.
OLIVEIRA, Denis Ernest Ligorio, B.E., of Bombay, India.
ONUOHA, Nicholas Onye Mauwa, of Birmingham.
PARSONS, Eric Henry, of Dartford, Kent.
PATEL, Vinodrai Khodabhai, B.Sc.(Eng.), of Eldoret, Kenya Colony.
PAUL, Iqbal Singh, of London.
PRASAD, Satrughan, B.Sc., of Patna, India.
PROUDFOOT, Thomas, of Hamilton, Lanarkshire.
PUN KIM CHUEN, of London.
RAHA, Jogabrata, of Hounslow, Middlesex.
RIZVI, Aulad Hussain, B.Sc., of London.
SAAFAN, Saafan Abdel-Gawad, B.Sc., of Manchester.
SAFRI, Taiyab Lookmanbhai, B.E., of Ahmedabad, India.
SCHROEDER, Werner Otto Esmarch, of Johannesburg, South Africa.
SESHAGIRI RAO, Kompella Venkatu S., of Chembur, Bombay, India.
SHARMA, Har Bhagwan, B.A., of Bhopal, India.
SIGERSON, Henry, of Glasgow.
SIM, Donald Hugh, B.E., of Wellington, New Zealand.
SMITH, Ian Charles, M.E., of Oxted, Surrey.
SMITH, John Nicoll, of Stockport, Cheshire.
SMITH, Peter William, of Banstead, Surrey.
TANG CHI KIN, of London.
THOMAS, Warwick, of North Sydney, New South Wales.
VISWANATHAN, Thellur Balasubramaniam, B.E., of West Bengal, India.
TURNER, John Robert, of Birmingham.
WAN, Kwan-Cheung Thomas, of London.
WATSON, John, of Broxbourne, Herts.
WATSON, Victor, of Enfield, Middlesex.
WHITBY, Laurence Peter, of Watford, Herts.
WRIGHT, James, of Kenya, East Africa.
WU, Gordon Ying Sheung, B.Sc., of Hong Kong.
YATES, Gordon, of London.
YU CHUNG YEUNG, of London.
YUNG, Oliver Hon Bun, of Hong Kong.

ASSOCIATE-MEMBERS

BLASZYK, Tadeusz, B.Sc., of London.
BUNN, David Michael, of Sanderstead, Surrey.
CHOPRA, Narinder Nath, of Burdwan, West Bengal.
CLEMENTS, Alfred Cecil, of Southampton.
CORRY, John Percival, of Orpington, Kent.
COYNE, Terence, B.Sc., of London.
CROSS, John Henry, of Chessington, Surrey.
FAIRWEATHER, Roy Leonard, of Broxbourne, Herts.
FARSIDES, Costas Pavlou, of London.
GRIEVE, Neil Macpherson, of Reading, Berks.
HARRIS, Martin Anthony, B.Sc., of Newcastle-on-Tyne.
HARRIS, Peter William, of Caterham, Surrey.
HOLTUM, Ronald David, of London.
HUFFORD, Howard Horace, A.M.I.C.E., of Singapore.
JAMES, Harold Cecil, M.A., A.M.I.C.E., of Banstead, Surrey.
KING, Peter Henry, of London.
LAWSON, Harland, of London.
LIEBNER, Rolf Otto, of Windhoek, South West Africa.
MCLEAN, Robert, B.E., of Cairns, Queensland, Australia.
MADIN, Carl Lionel, of Werrington, Peterborough.
MAHAJAN, Krishnaji Dattatray, B.Sc., B.E., of Poona, India.
MICHNA, Tadeusz, of London.
MILLIGAN, Owen Page, of Beaconsfield, Bucks.
OXLEY, Victor John, of London.
ROBINSON, Frank Leonard, B.Sc., of London.
ROBINSON, George Leslie, B.Sc., of Banstead, Surrey.
TURNBULL, James Neil, B.E., of Palmerston North, New Zealand.

WELSH, John Sturgeon, of West Hartlepool, Co. Durham.
WHITE, Stanley Francis, of London.
WILIMOWSKI, Zbigniew Marian, of London.
WILSON, Frederick, of Corstorphine, Edinburgh.
WILSON, Norman Angus, A.M.I.C.E., of Cheam, Surrey.

TRANSFERS

Students to Graduates

ADESOGA, Mobolaji, of London.
CATO, Ross Edward, of Auckland, New Zealand.
CHIA CHER KENG, of Singapore.
DAS, Bishnu Pada, B.Sc., of London.
FEEROZ, Dawoodi Zainul, of Bombay, India.
GAJJAR, Chandrakant Damji, B.Sc., of Richmond, Surrey.
GOULD, Charles James, of London.
HUGHES, Peter James, of London.
LAI SHUI KWAN, of Hong Kong.
LEE CHEOK HOE, of Singapore.
LOCKE, Joseph, of Bolton, Lancs.
MCCARTAN, Peter John, of Port Elizabeth, South Africa.
MILLER, Roy James, of Bolton, Lancs.
MOORES, Arthur Keith, of Bolton, Lancs.
OGUNSOLA, Thompson Olusola, of London.
OKAFOR, Ebenezer Chikwudozie, of London.
POTGIETER, Theodorus Daniel, of Transvaal, South Africa.
REID, Michael Irwin, of Belfast, Northern Ireland.
SALAKO, James Adebayo, of London.
SHASTRI, Madhusudan Keshavram, B.E., of Ahmedabad, India.
SIVALINGAM, Gunaratnam, of Colombo, Ceylon.
SMITH, James Myles, of Motherwell, Lanarkshire.
TANG HUNG CHUN, of Hong Kong.
TOWLER, Malcolm Philipson, of Liverpool.
WHELAN, Peter Denis, of London.
WRIGHT, Brian Allen, of Johannesburg, South Africa.
WROE, Herbert Brian, of Harrogate, Yorks.

Student to Associate-Member

GROVE, James Arthur, of Bromley, Kent.

Graduates to Associate-Members

ANDERSON, David Graham, A.M.I.C.E., of Cardiff, Glam.
BEAMAN, Jack, M.Sc., of Heald Green, Cheshire.
BENNETT, John David, B.Eng., of London.
BHOGAL, Harbans Singh, B.Sc.(Hons.), of Cardiff.
BISHOP, Colin Charles, B.Sc., of Cardiff.
BROWN, Roy Charles, of Dorking, Surrey.
BUSH, Kenneth Donald, of Rolleston on Dove, Nr. Burton on Trent, Staffs.
CLARKE, Francis Gerard, of Radcliffe, Lancs.
CLOUGH, Norman, of Newton, Chester.
CORBETT, Brian Oliver, B.Sc., A.M.I.C.E., of London.
CRUDDAS, Edward Alan, B.Sc., of Beckenham, Kent.
DODD, Harry Thomas, of Sutton Coldfield.
DOUET, Louis Maurice, of London.
DOUGLAS, James Pollock Donaldson, of Edinburgh.
DRAKE, Michael John, of Cwmbran, Mon.
DUTT, Nihar Kusum, B.Sc., A.M.I.C.E., of London.
ELLISON, Morville Vernon, of Bishops Stortford, Herts.
EVANS, David Geoffrey Rhys, B.Sc.(Eng.), of Ashford, Middlesex.
FRIBBANCE, William Laurence, of London.
GHOSH, Srikumar, B.E., of London.
GILLIBRAND, William, B.Eng., of London.
GRAY, Bruce Alan, of Leigh-on-Sea, Essex.
HADDOCK, Geoffrey, of Weaverham, Nr. Northwich, Cheshire.
HAGENBACH, Paul, of London.
HALL, Conrad, of Bottesford, Lincs.
HALLETT, Rex, of Harrow Weald, Middlesex.
HARRIS, Clifford James, of Port Elizabeth, South Africa.
HASTINGS, Joseph William, of London.
HERON, James Alexander Stewart, of Johannesburg, South Africa.
HOJEM, Peter Anthon, B.Sc., of Lusaka, Northern Rhodesia.
HOLROYD, Philip, A.M.I.Mech.E., of Slaithwaite, Nr. Huddersfield.
HUNT, David Charles Gowar, of Hamble, Hants.
INGRAM, Leonard Victor Walter, of London.
JORDAN, Edward John, of Cheadle, Cheshire.
KERNS, Donald Patrick, of London.
LAWRENCE, Hedley, of Vereeniging, South Africa.
LAXTON, David James, of Harlow, Essex.
LI KUI WAI, B.Sc.(Eng.), of Hong Kong.
LIPP, Lawrence, of Stanmore, Middlesex.
LOUGHTON, Alan James, of Bexley, Kent.
LOVELESS, Colin, B.Sc., A.M.I.C.E., of Angmering Green, Sussex.
LUHAR, Jashbhai Dahyabhai, B.E., of Dar-es-Salaam, British East Africa.
MCGRATH, Frederick James, B.Sc., of Richmond, Surrey.
MCKAY, Colin, A.M.I.Mun.E., of Shrewsbury.
MATHEW, George, of Rutherglen, Scotland.
MILLAR, Malcolm Alexander, of Altrincham, Cheshire.
NURSE, Colin Frederick, of Scunthorpe, Lincs.
NURSEY, John Richard, of Norwich.
ORENSTAIN, Jehuda Leo, B.Sc., of London.
PATTERSON, Robert Alfred, of Dunston, Gateshead.
PILGRIM, Ernest Joseph, of Twickenham, Middlesex.
PONTIN, Ronald Frederick, of Harlow, Essex.
RAWSON, Morris, of Newton, Derbyshire.
RICKETTS, Daniel John, of Bickley, Kent.
SEN GUPTA, Pranab Kumar, B.E., of Bihar, India.
SMITH, David Harry, of Ipswich, Suffolk.
SMITH, John Frank McLaren, of East Grinstead, Sussex.
SPICER, Douglas Aubrey, of Johannesburg, South Africa.
STONE, Peter Albert, of Ruislip, Middlesex.
SYMES, Ronald Clive, of Wickford, Essex.
THOMSON, John Stuart, B.Sc.(Eng.), of Kingston, Jamaica.
TURNER, Frank Herbert, of Croydon, Surrey.
WALKER, Derek, of Yeadon, Nr. Leeds, Yorks.
WALLACE, Reginald Rutherford, B.A.Sc., of London.
WHITE, Keith Christopher, B.Sc., of Kidlington, Oxford.
YOUNG, James, of Salfords, Nr. Redhill, Surrey.

Associate-Members to Members

BARNES, William Gordon, of Adelaide, South Australia.
HERSON, Sydney, of Wilmslow, Cheshire.
JENKINS, Arthur Henwood, B.Sc., A.M.I.C.E., of Dringhouses, York.
MARSTON, George Howard, B.Sc.(Tech.), A.M.I.C.E., of Marple, Cheshire.
RAMAGE, Lanes Morton, B.Sc., M.I.C.E., of London.
WEISS, Charles, of London.

Associate-Member to Retired Associate-Member

HAMER, Llewellyn Clifford, O.B.E., of London.

OBITUARY

The Council regret to announce the deaths of David William BRISBANE, and Arthur BROCKLEHURST (Members) Major Leslie SHINGLETON (Retired Member), Norman Frederick WHEELER (Associate-Member).

RESIGNATION

Notice was given that the Council had accepted with regret the resignation of Archibald Augustus SMITH (Retired Associate-Member).

WITHDRAWAL OF RESIGNATION —REINSTATEMENT

The resignation of the following member has been withdrawn : Charles Hugh BELL (Associate-Member).

JUNE MEETINGS

Thursday, 8th June, 1961

An Ordinary Meeting of the Institution of Structural Engineers will be held jointly with the British Section of the Société des Ingénieurs Civils de France at 11, Upper Belgrave Street, London, S.W.1., on Thursday, 8th June, 1961, at 6 p.m., when a paper entitled "Pathology of Construction" will be given by M. Albert Brenier, Director of the Société pour la Controle Technique et Expertise de la Construction and Delégué Général of the Bureau Securitas.

Thursday, 22nd June, 1961

An Ordinary General Meeting of the Institution of Structural Engineers will be held at 11, Upper Belgrave Street, London, S.W.1, on Thursday, 22nd June, 1961, at 5 p.m. This meeting is for the election of members and is open only to corporate members of the Institution.

COMMONWEALTH TECHNICAL TRAINING WEEK

The exhibition of photographs, models and drawings illustrating the profession of Structural Engineering which the Institution is staging at 11, Upper Belgrave Street, London, S.W.1., closes at 8 p.m. on the 2nd June.

The exhibition is open daily from 10 a.m. to 12 noon and from 2 p.m. to 8 p.m. Members of the Institution are available to answer questions regarding training for the profession and prospects.

It is hoped that members and their friends who are able to do so will visit the exhibition, particularly between 6 and 8 o'clock each evening.

EXAMINATIONS, JULY, 1961

The Examinations of the Institution will next be held in the United Kingdom and overseas on Tuesday and Wednesday, 11th and 12th July, 1961 (Graduateship), and Thursday and Friday, 13th and 14th July, 1961 (Associate-Membership).

PRIZES—JANUARY, 1961, EXAMINATIONS

The Council have awarded the following prizes in respect of examinations held in January, 1961 :—

ANDREWS PRIZE.—For the candidate who obtains the highest aggregate of marks in the Associate-Membership Examination, passing in all subjects.
D. P. KERNS, of London.

HUSBAND PRIZE.—For the candidate who takes the whole of the Associate-Membership Examination, passes in all subjects, and obtains the highest marks in the paper "Structural Engineering Design and Drawing."
S. H. ABU-SITTA, of London and D. A. SPICER, of Johannesburg.

WALLACE PREMIUM (SENIOR).—For the candidate who takes the whole of the Associate-Membership Examination, passes in all subjects, and obtains the highest marks in the paper "Theory of Structures (Advanced)."
J. LOCKE, of Bolton.

A. E. WYNN PRIZE.—For the candidate who takes the whole of the Associate-Membership Examination, passes in all subjects, and obtains the highest marks in the Reinforced Concrete Section 'C' of the paper "Structural Engineering Design and Drawing."
S. H. ABU-SITTA, of London and D. A. SPICER, of Johannesburg.

GRAHAM WOOD PRIZE.—For the candidate who obtains the highest marks in the Steel Section 'B' of the paper "Structural Engineering Design and Drawing" in the Associate-Membership Examination.
A. C. CLEMENTS, of Southampton.

WALLACE PREMIUM (JUNIOR).—For the most successful candidate in the Graduateship Examination, passing in all subjects.
K. B. HINGORANI, of London.

INSTITUTION AWARDS

The Council have awarded Institution Bronze Medals for the Session 1959-60 to Dr. J. D. Geddes and Mr. D. W. Cooper for their paper "Structures in Areas of Mining Subsidence," and to Mr. T. C. Waters for his paper "The Structural Engineer in the Field of Atomic Energy."

The Institution Branch Award (for the best paper from amongst those for which Branch Prizes have been awarded) has also been given to Dr. J. D. Geddes and Mr. D. W. Cooper.

REPUBLIC OF IRELAND SECTION

The Council have sanctioned the formation of a Republic of Ireland Section of the Institution with headquarters in Dublin. The Honorary Officers and Committee members are as follows :—

Chairman : Mr. B. J. Dixon, B.Sc., A.I.E.E., A.M.I.Mech.E. (Member)

Honorary Secretary : Mr. P. J. Carroll, M.E., A.M.I.C.E., A.M.I.C.E.I. (Associate-Member).

Committee Members : Messrs. J. D. Medcalf, B.A., B.A.I., A.M.I.C.E.I., R. L. Smith, B.A., B.A.I., A.M.I.C.E.I., M. B. Gill, B.E., T. J. Powell, T. F. Russell, B.E., B.Sc., B. Le Cesne Byrne, B.E., A.M.I.C.E.

Communications should be addressed to Mr. P. J. Carroll, M.E., A.M.I.Struct.E., A.M.I.C.E., A.M.I.C.E.I., 9, Laburnum Road, Clonskea, Dublin, Ireland.

REPRESENTATION

The following Institution representatives have been appointed :—

West Midlands Advisory Council for Technical Education, Civil Engineering Advisory Committee :
Mr. W. H. VEAL (re-appointed)

Northern Advisory Council for Further Education, Advisory Committee for Building :
Mr. E. ATKINSON (to succeed Mr. C. A. HARDING)

BSI Cement, Lime and Gypsum Products Industry Standards Committee CEB/-
Mr. K. SEVERN (to succeed Mr. LESLIE TURNER)

BSI Civil Engineering (Universal Decimal Classification 624/628) Panel, OC/20/4/16.
Mr. K. SEVERN (to succeed Mr. LESLIE TURNER)

CONFERENCE ON TIMBER ENGINEERING

The First International Conference on Timber Engineering organised jointly by the University of Southampton and the Timber Development Association, is to be held at the University of Southampton, from September 18th-23rd, 1961.

Papers will be presented to the Conference under four main headings—description, construction, design, research. Subjects covered will include "Shell Roofs

in Brussels and Holland" by J. H. Pestman of Holland; "The Design, Testing and Construction of a Timber Conoid Shell Roof" by Dr. L. G. Booth of the University of Southampton; "The application of research to the design of timber roof structures" by D. W. Cooper of the University of Durham; as well as papers on the economics of timber roofing, international development in the design and construction of timber shells and folded plates, methods of jointing and testing of timber roof structures. Part of the conference will be devoted to site visits.

The Conference Secretary is Dr. H. G. Allen, Civil Engineering Department, The University, Southampton, from whom further information may be obtained.

ADDITIONS TO THE LIBRARY

AMERICAN INSTITUTE OF STEEL CONSTRUCTION. *National Engineering Conference, 1960, Proceedings.* New York, 1960.

BRESLER, B. and LIN, T. Y. *Design of Steel Structures*, New York & London, 1960. Presented by Mr. W. Bates.

CASE, J. and CHILVER, A. H. *Strength of Materials*, London, 1959.

CEMENT & CONCRETE ASSOCIATIONS. *Standard beam sections for prestressed concrete bridges*, London, 1961.

ENGINEERING INSTITUTE OF CANADA. *Engineering Careers in Canada*, 1960-61. Montreal.

FLÜGGE, W. *Stresses in Shells*, Berlin, 1960. Presented by Mr. A. R. Dykes. *3rd Japan Congress on Testing Materials Proceedings*, Kyoto, 1960.

JASTRZEBSKI, Z. D. *Nature and Properties of Engineering Materials*. New York & London, 1959. Presented by Mr. J. B. Story.

JOHNSON, A. and KING, W. H. *A Manual of Specifications and Quantities for Civil Engineers, 5th Ed.* London, 1957.

LEWITT, E. H. *Hydraulics and Fluid Mechanics, 10th Edition*, London, 1958.

MURDOCK, L. J. *Concrete Materials and Practice, 3rd Edition*, London, 1960.

NATIONAL BUREAU OF STANDARDS RESEARCH HIGHLIGHTS. *Annual Report*, 1960., Washington, D.C., 1960.

NOVOZHILOV, V. V. *The Theory of Thin Shells*, Groningen, The Netherlands, 1959. Presented by Mr. F. G. Alchin.

PAYNE, A. R. and SCOTT, J. R. *Engineering Design with Rubber*, London, 1960. Presented by Mr. D. R. R. Dick.

PEABODY, D. Jr. *Design of Reinforced Concrete Structures, 2nd Edition*, New York & London, 1946.

ROARK, R. J. *Formulas for Stress and Strain, 3rd Edition*, New York & London, 1954.

ROSLI, A. *The Dynamic Behaviour of Prestressed Bridges*, C. & C.A. Library Translation, No. 93, 1960.

TIMBER DEVELOPMENT ASSOCIATION. *Test Records B/TR/3, The National Resistance of Certain Timber Species to Marine Borers., E/TR/18. Load Tests on TDA Lamp Standards. Test Memorandum E/TM/40 Comparative Test on the Nailing properties of European redwood and western red cedar.*

The following volumes have been presented by Mr. H. G. Wuest, M.I. Struct.E.

BECK, E. G. *Tank Construction. Relating principally to the Design. Manufacture and Erection of Tanks in Mild Steel*, London, 1921.

BIJLAARD, P. P., KOLLBRUNNER, C. F. and STUSSI, F. *Theory and tests concerning buckling of plates stressed by equally distributed longitudinal stresses.(In German). International Association for Bridge and Structural Engineering*, Third Congress, Preliminary Publication, 1948, Reprint.

KOLLBRUNNER, C. F. *Weitgespannte Hallen aus Stahl*, Zürich, July, 1948.

KOLLBRUNNER, C. F. and HERMANN, G. *Elastische Beulung von auf einseitigen, ungleichmassigen Druck beanspruchten Platten*. Zürich, 1948.

THE LINCOLN ELECTRIC COMPANY. Cleveland, Ohio, U.S.A. *Procedure Handbook of Arc Welding Design and Practice, 6th Edition*, Ohio, 1940.

MARSH, C. F. and DUNN, W. *Manual of Reinforced Concrete, 3rd Edition*, London, 1920.

MATTHEWS, E. R. *Studies in the Construction of Dams* London, 1919.

THE MINISTRY OF WORKS. *The Standard of Wartime Building*, London, 1943.

RICHART, F. E. and OTHERS. *The effect of eccentric loading, protective shells, slenderness ratios, and other variables in reinforced concrete columns*, University of Illinois, 1947.

WOLFE, W. S. *Graphical analysis. A text book on Graphic Statics*, New York & London, 1921.

The following Paper has been added to the Library :—
"*The Degree of Redundancy of Structures*" by K. C. Rockey, M.Sc., Ph.D., A.M.I.C.E., A.M.I.Mech.E., and B. W. Preece, B.Sc.

CHANGE OF TELEPHONE NUMBER

The telephone number of the Institution has been, changed to BELgravia 4535 (3 lines).

YEAR BOOK AND LIST OF MEMBERS

The Year Book and List of Members for 1961 will go to press in July, for publication in October, when a copy will be sent to all members.

Members are requested to inform the Secretary of any alterations in titles, degrees or addresses, which have not already been notified, by June 24th, in order that such amendments may be included in the new edition.

RECENT PUBLICATIONS OF INTEREST TO STRUCTURAL ENGINEERS

ADMIRALTY

Admiralty Advisory Committee on Structural Steel. (Chairman, A. J. Sims, O.B.E., R.C.N.C.). A Comparison of Transition Temperatures determined by Small and Large Tests on Five Steels. Report No. P.2 (20-165-2). 9s. 6d.

MINISTRY OF EDUCATION

Better Opportunities in Technical Education. (Cmnd. 1254). 1s. 5d.

Selected Statistics relating to Local Education Authorities in England and Wales. Educational Year 1958-59. List 71 (1958-59) (27-36-0-59). 3s. 4d.

DEPARTMENT OF HEALTH FOR SCOTLAND

Housing Summary. November 30, 1960. (Cmnd. 1259). 5d.

Technical Memorandum 1, *Slender Wall Construction for Houses.* Revised Edition, 1960. 3s. 4d.

MINISTRY OF HEALTH

Hospital Building Note No. 1, *Buildings for the Hospital Service.* (32-498-1). 1s. 8d.

MINISTRY OF LABOUR

Accidents at Factories, Docks, Building Operations and Works of Engineering Construction. How they Happen and how to Prevent them. No. 46, January, 1961. (72-21-46). 1s. 7d.

Choice of Careers No. 92. *Professional Engineers.* Second Edition, October, 1960. (36-170-92-60). 2s. 1d.

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH

Building Research Station Digest No. 6 (2nd Series). *Drainage for Housing.* (72-36-0-6) 6d.

Building Science Abstracts. Vol. XXXIII, 10th October, 1960. Abstracts Nos. 1557 to 1724. (72-4-60-10). 3s. 4d.

11th November, 1960, Abstracts Nos. 1725 to 1905. (72-4-60-11). 3s. 4d.

D.S.I.R. Research Grants, 1961. (47-220-0-61). 1s. 2d.

D.S.I.R. Studentships and Fellowships, 1961. (47-158-0-61). 2s. 1d.

Factory Building Study No. 9, *Fire Protection in Factory Buildings* (by G. J. Langdon-Thomas, A.A.Dip., A.R.I.B.A.), October, 1960. (47-212-9). 3s. 10d.

Forest Products Research Bulletin No. 45, *Strength Properties of Timber, Addendum,* Gratis.

Fuel Research 1917-1958, *Review of the Work of the Fuel Research Organisation of D.S.I.R.* (47-225). 15s. 9d.

Hydraulics Research Paper No. 3, *Coast Erosion and Defence.* Nine questions and answers (by R. C. H. Russell, M.A., A.M.I.C.E.), October, 1960. (47-207-3). 1s. 5d.

Road Abstracts Vol. XXVII, No. 10, October, 1960. Abstracts Nos. 974 to 1098. (72-25-60-10). 2s. 8d.

SCOTTISH EDUCATION DEPARTMENT

Technical Education in Scotland, Pattern for the Future. (Cmnd. 1245). 1s. 2d.

UNITED KINGDOM ATOMIC ENERGY AUTHORITY

Development and Engineering Group. D.E.G. Report 120(*R*) *Uranium Ceramics Data Manual. Properties of Interest in Reactor Design.* (Compiled by B. J. Seddon). March 7th, 1960. 16s. 9d.

D.E.G. Report 219 (*C*), *Determination of Aluminium in Pure Uranium and in Aluminium-Uranium Alloys.* (Oxine Spectrophotometric Procedure). (By C. O. Granger), August 24th, 1960. 2s. 10d.

D.E.G. Report 229 (*CA*), *Estimation of Surface Areas by a Gas-Chromatographic Method* (by J. F. Ellis, C. W. Forrest, D. D. Howe), August 19th, 1960. 2s. 1d.

UNITED NATIONS. (UN)

European Housing Trends and Policies in 1959. (60.II.E.Mim.11). 5s. 7d.

European Steel Market in 1959. (60.II.E.Mim.10). 7s. 9d.

Housing and Building Statistics for Europe, Annual Bulletin, 1959. (60.II.E.5). 5s. 6d. Quarterly Bulletin, Vol. VIII, No. 2, 1960. 4s. 8d.

PERIODICALS NON-PARLIAMENTARY

Timber Bulletin for Europe (Quarterly 7s. 9d., plus Supplements, £1 16s.)

Road Abstracts (monthly 2s. 8d.). £1 13s.

Quarterly Bulletin of Steel Statistics for Europe (7s. 8d.). £1 5s.

Journal of Nuclear Fusion (Quarterly £3 10s.).

TIMBER DEVELOPMENT ASSOCIATION

Test Record E/TR/18, Load Tests on T.D.A. Lamp Standards. Published by The Timber Development Association Ltd., 21, College Hill, London, E.C.4.

Test Memorandum E/TM/40, Comparative Test on the Nailing Properties of European Redwood and Western Red Cedar. Published by T.D.A.

Branch Notices

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